(2) Interactions of high energy particles with matter

- Ionization
- Bremsstrahlung
- Cherenkov radiation
- Nuclear Interactions
  - interaction with magnetic fields: Synchrotron radiation
Ionization

High energy particles have different effects on the matter (solid, liquid, gas) they traverse:
- Destruction of crystal structures and molecular chains.
- Nuclear interactions with the nuclei of the material.
- Ionisation and excitation of atoms and molecules of the material.
  -> source of charge and heat in the material

In H.E. Astrophysics, ionization is very important for several reasons:
- Heating mechanism for interstellar gas.
- Influence on propagation of high energy particles.
- In detectors, ionisation can be used to measure flux and properties of high energy particles.
Ionization - Non-relativistic approximation of dE/dx (1)

Assumptions:
- From simple collision picture: If $M \gg m_e$ the incident high energy particle is effectively undeviated.
- H.E. particle moves sufficiently fast that we can take the position of the electron in its orbit to be stationary during interaction.

1.) **total kinetic energy transferred to one electron**
(forces parallel to line of flight cancel out)

\[
F_p = \frac{ze^2}{4\pi \varepsilon_0 r^2} \sin(\theta) \quad \rightarrow \quad p = \int F_p \, dt = \int_{-\infty}^{\infty} \frac{ze^2}{4\pi \varepsilon_0 r^2} \sin(\theta) \frac{1}{v} \, dx;
\]

with \( \frac{b}{x} = \tan(\theta) \), \( r = \frac{b}{\sin(\theta)} \), \( dx = \left(\frac{-b}{\sin^2(\theta)}\right) \, d\theta \);
Ionization - Non-relativistic approximation of $dE/dx$ (2)

$$p = -\int_{0}^{\pi} \frac{ze^2}{4\pi \varepsilon_0 b^2} \sin^2(\theta) \frac{b \sin(\theta)}{v \sin^2 \theta} \, d\theta = -\frac{ze^2}{4\pi \varepsilon_0 b v} \int_{0}^{\pi} \sin(\theta) \, d\theta$$

$$p = \frac{ze^2}{2\pi \varepsilon_0 b v} \rightarrow E_{\text{kin}} = \frac{p^2}{2m_e} = \frac{z^2 e^4}{8\pi^2 \varepsilon_0^2 b^2 v^2 m_e}$$

2.) **average energy loss per unit length** (-$dE/dx$)

If $N_e$ is the electron density in the material, the number of electrons in a cylindric shell around the x-axis of length $dx$, inner radius $b$ and outer radius $b+db$ is:

$$N_e \cdot 2\pi b \, db \, dx \rightarrow -dE = \int_{b_{\text{min}}}^{b_{\text{max}}} N_e \, 2\pi b \, db \frac{z^2 e^4}{8\pi^2 \varepsilon_0^2 b^2 v^2 m_e} \, dx$$

$$\frac{-dE}{dx} = \frac{z^2 e^4 N_e}{4\pi \varepsilon_0^2 v^2 m_e} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)$$

3.) **determination of the upper and lower limit of $b$**

**upper limit $b_{\text{max}}$:**

If $b$ is so large that the duration of the collision is greater than the period of the electron's orbit, the interaction is no longer impulsive and no ionisation takes place.
First we need an estimate for the duration of the collision:

\[ p = \int_{-\infty}^{\infty} F_p \, dt \approx F[b] \, \tau \quad ; \quad \tau \approx \frac{2b}{v} \]

Then, \( b \) is a good (rough) estimate of \( b_{\text{max}} \) when the duration of the collision is equal to the orbital period of the electron:

\[ \tau_{\text{max}} \approx \frac{2b_{\text{max}}}{v} \approx \frac{1}{v_0} \quad \rightarrow \quad b_{\text{max}} \approx \frac{v}{2v_0} = \frac{v \pi}{\omega_0} \]

We can express the electron's orbital frequency in terms of a binding energy or ionization potential of the atom. Electron behaves like oscillator with:

\[ I = \frac{1}{2} \hbar \omega_0 \]

In reality, we have to consider electrons at many different energy levels. We need thus a properly weighted mean \( \langle I \rangle \). This is a characteristic of the material and has to be measured. \( \langle I \rangle \) is called the effective ionization potential.

Thus:

\[ b_{\text{max}} \approx \frac{v}{2I} \approx \frac{v \hbar}{\langle I \rangle} \]

**lower limit \( b_{\text{min}} \):**

Here we use Heisenberg's Uncertainty Principle and the fact that the maximum velocity gained by the electron is about \( 2v \). (This is the result of a central elastic collision with \( M >> m_e \).)
Ionization - Non-relativistic approximation of $dE/dx$ (4)

$$\Delta x \Delta p \approx \hbar ;$$
$$\Delta v \approx 2v \rightarrow \Delta p \approx 2m_e v \rightarrow \Delta x \approx \frac{\hbar}{2m_e v} ;$$

$$b_{\text{min}} \approx \frac{\hbar}{2m_e v}$$

=> Non-relativistic approximation for the energy loss rate due to ionization:

$$- \frac{dE}{dx} = \frac{z^2 e^4 N_e}{4\pi \varepsilon_0^2 v^2 m_e} \ln \left( \frac{2m_e v^2}{\langle I \rangle} \right)$$

- Ionization losses are independent of the mass of the high energy particle.

- They are proportional to the $(z/v)^2$ of the h.e. particle.
  --> important for measurement of particle characteristics.

- They are indirectly proportional to the electron mass $m_e$.
  --> the effect of the electrostatic interaction on protons and nuclei is negligible.

- Logarithmic dependence on the kinetic energy of the high energy particle.

- Logarithmic dependence on $1/\langle I \rangle$, a characteristic of the material.
Ionization - Bethe Bloch formula

Exact Derivation of the ionization energy loss formula using relativistic quantum theory leads to:

\[- \frac{dE}{dx} = \frac{z^2 e^4 N_e}{4 \pi \varepsilon_0^2 v^2 m_e} \ln \left( \frac{2 \gamma^2 m_e v^2}{\langle I \rangle} - \frac{v^2}{c^2} \right)\]  (Bethe–Bloch formula)

up to \( E_{\text{kin}} \approx M_c^2 \):
\[ \frac{dE}{dx} \sim \frac{1}{v^2} \sim \frac{1}{E} \]

for higher energies:
\[ \frac{dE}{dx} \sim \log(\gamma^2) \]

minimum loss rate for \( E_{\text{kin}} \approx M_c^2 \) particles of \( \beta \gamma \) between \( \sim 1 \) and \( 10 \) are called «minimum ionizing». In practice, most relativistic particles have mean energy loss rates close to the minimum (e.g. cosmic ray muons)

from:
Ionization - Practical forms of the B.-B. formula

We rewrite the B.-B. formula for practical use:
- if the atomic number of the medium is $Z$ and the number density of atoms is $N$, then $N_e = N \cdot Z$.
- If the density of the medium is $\rho$ and its atomic mass $m$, then $N_e = Z \cdot \frac{\rho}{m}$
- we express the ionization loss not in terms of the pathlength $dx$, but in terms of the total mass per unit cross-section (grammage) traversed by the high energy particle: $dx \rightarrow d\xi = \rho \cdot dx$ (in units g/cm²)

\[
- \frac{dE}{d\xi} = \frac{z^2 e^4 Z}{4 \pi \varepsilon_0^2 v^2 m_e m} \ln \left( \frac{2 \gamma^2 m_e v^2}{\langle I \rangle} - \frac{v^2}{c^2} \right) = z^2 \frac{Z}{m} f(v, \langle I \rangle)
\]

- $Z/m$ varies very little for all stable elements. Thus the ionization loss rate depends only on:
  - the charge of the high energy particle ($z$)
  - the velocity of the high energy particle ($v$)
  - the ionization potential of the medium ($\langle I \rangle$)

Ionization loss is smaller for materials with larger ionization potential $\langle I \rangle$ (the large difference with H₂ and He is due to the "density effect", which we will not discuss here)
The range that a h.e. particle traverses in the medium gives an estimate of its total initial energy. It is found by integrating over its energy loss rate. A rough estimate for the range of relativistic particles is given by dividing their energy by the minimum ionization energy:

\[ R \sim \frac{E_{\text{kin}}}{(dE/dx)_{\text{min}}} \]

Note that \((dE/dx)_{\text{min}}\) is proportional to \(z^2\)!

More careful calculations allow to define the range as a function of the energy for different incident particles as seen on the right.
Application in Detectors

- cloud chamber
- bubble chamber
- ionisation chamber

“He hasn’t been himself ever since he got that new bubble chamber!”
The Cloud Chamber (1)

- First cloud chamber developed in 1911 by Charles Wilson. 1932 Carl Anderson discovers positron.

- A cloud chamber is a closed container with supersaturated vapour (e.g. water in air). An ionizing particle that traverses the chamber leaves a trail of charged particles that serve as condensation centers. Different particles at different energies can thus be distinguished by the thickness and length of the visible condensation tracks. Applying a magnetic field allows to distinguish between different charges.
The Cloud Chamber (2)

- **Wilson cloud chamber** ("pulsed type"): The air inside a sealed chamber is saturated with water vapour. The pressure inside the chamber can be suddenly reduced, which cools the air and leads to supersaturation.

- **Diffusion cloud chamber**: The air inside the chamber is saturated with alcohol vapour. A large temperature difference is maintained between the top and bottom of the chamber. The air-vapour mixture cools as it diffuses toward the cool bottom and becomes supersaturated. Continuous operation possible.
The Bubble Chamber (1)

- Invented 1952 by Donald Glaser. Similar concept as cloud chamber.

- A bubble chamber is a vessel filled with superheated transparent liquid (e.g. hydrogen). An ionising particle deposits energy and causes the liquid to boil along its track. A trail of bubbles of vaporised liquid becomes visible.

- Usually, a liquid is brought just below boiling point. A rapid decrease in pressure brings it to a superheated state.
The magnetic field causes charged particles to travel helical paths whose radius is determined by the ratio of momentum to charge of the particle. In this way charged particles can be observed and their mass can be measured, if one has an estimate of their velocity.

$$qvB = mv^2/r \implies r = mv / (qB)$$

Cloud and bubble chambers have largely been replaced by wire chambers and spark chambers, which allow also a measurement of the particle energy. Bubble chambers have gained new interest for dark matter searches.
The Ionization Chamber

- When an ionizing particles pass through a gas it creates ion pairs. One method of measuring the ionisation loss $dE/dx$ is to collect the ions produced. There are three important instruments which use this concept: ionization chambers, proportional counters, and Geiger counters.

- The ionization chamber can be filled with a gas or liquid or be open to the atmosphere. A high voltage is applied to the anode wire, the cathode housing is connected to ground. Radiation or ionising particles will create ion-electron pairs. The positive ions will drift towards the cathode and the electrons towards the anode in the center of the tube. The freed charge can be measured as a voltage.

- Other than Cloud and Bubble Chambers, Ionization Chambers are sensitive to high energy photons.

- Ionization chambers are not very sensitive to a single minimum ionizing particle, more useful for heavy and slow ions. The response time is relatively large.

The different types of gas-filled detectors will be discussed in more detail in the next chapter.
Nuclear Emulsions

- The method of nuclear emulsions was developed by Cecile Powell, who used it to discover the pion in 1947.

- An emulsion is made, as for photographic film, of a silver salt, usually bromide, embedded in gelatine and spread thinly on a substrate. Multiple layers of emulsion were historically the first means of visualizing charged particle tracks by their ionization. Emulsion detectors were carried on balloons to detect cosmic rays.

- Emulsion stacks are still used today to record, with very high positional precision (about one micron), very short tracks (e.g. tau leptons, which have a track length of less than a millimetre), or in other circumstances demanding very high precision.

- Emulsions are permanently sensitive and cause nontrivial data acquisition work by microscopic methods. Usually, emulsions are left in place for long runs, and hence are restricted to applications in areas of small particle flux or in low-cross-section experiments, like neutrino physics. Data acquisition by automated means (e.g. by scanning the film with a CCD camera) has been found possible in some circumstances.

(based on the Particle Detector BriefBook)
In experiments in the 1930's, it was found that the ionization loss rate underestimates the energy loss rate for relativistic electrons.

An additional energy loss mechanism was found: «Bremsstrahlung» (or «free-free emission») is the electromagnetic radiation of electrons that are accelerated (or decelerated) in the electrostatic field (Coulomb field) of a nucleus.

Further significance for High Energy Astrophysics: hot ionised gas emits free-free radiation (radio emission from ionised hydrogen, X-ray emission from binaries and clusters of galaxies); bremsstrahlung is thus a mechanism for thermal emission.
Bremsstrahlung - some tools (1)

1.) The total radiation rate of an accelerated charged particle is given by the **Larmor formula**:

\[ P = - \left( \frac{dE}{dt} \right)_{\text{rad}} = \frac{q^2 |\vec{\dot{r}}|^2}{6\pi \varepsilon_0 c^3} \]

- radiation loss rate in the instantaneous rest frame of the particle
- radiated power $\sim \sin^2 \theta$. No radiation along the acceleration vector.
- radiation is polarised

2.) What is the spectral distribution of the emitted radiation?

Some tools will be useful:

a) Fourier transformation

\[ \dot{v}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \dot{v}(\omega) \exp(-i \omega t) \, d\omega \]

\[ \dot{v}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \dot{v}(t) \exp(-i \omega t) \, dt \]
b) Parseval's theorem
\[ \int_{-\infty}^{\infty} |\dot{\mathbf{v}}(\omega)|^2 \, d\omega = \int_{-\infty}^{\infty} |\dot{\mathbf{v}}(t)|^2 \, dt \]

c) If \( f(\omega) \) is a real function, then
\[ \int_{-\infty}^{\infty} |f(\omega)|^2 \, d\omega = 2 \int_{0}^{\infty} |f(\omega)|^2 \, d\omega \]

Thus the total emitted radiation can be written as:
\[ \int_{-\infty}^{\infty} dE = \int_{-\infty}^{\infty} \frac{e^2}{6\pi \varepsilon_0 c^3} |\dot{\mathbf{v}}(t)|^2 \, dt = \int_{-\infty}^{\infty} \frac{e^2}{6\pi \varepsilon_0 c^3} |\dot{\mathbf{v}}(\omega)|^2 \, d\omega = \int_{0}^{\infty} \frac{e^2}{3\pi \varepsilon_0 c^3} |\dot{\mathbf{v}}(\omega)|^2 \, d\omega \]

But we can also write the total emitted radiation as:
\[ \int_{0}^{\infty} I(\omega) \, d\omega \]

Thus:
\[ I(\omega) = \frac{e^2}{3\pi \varepsilon_0 c^3} |\dot{\mathbf{v}}(\omega)|^2 \]
Bremsstrahlung - non-relativistic limit (1)

We skip the lengthy calculation of $\hat{\nu}(\omega)$.

For an interaction with a single nucleus, the bremsstrahlung spectrum is constant for low frequencies and falls off exponentially at cutoff frequency $\omega_{\text{cut}} = v / b$.

To motivate this expression, remember that the duration of the collision is:

$$\tau \approx \frac{2b}{v} \rightarrow \omega \approx \frac{2\pi}{\tau} \approx \pi \frac{v}{b}$$

This duration determines the highest frequency that is emitted. For lower frequencies, the collision time is smaller than the period of the emitted lightwave. The collision occurs approximately instantaneous in time

-> Fourier transform is flat!

Similar to our ionization calculation, we have to integrate over all nuclei in a given volume around the path of the electron to get the total $I(\omega)$.
Bremsstrahlung - non-relativistic limit (2)

After calculating the acceleration of the electron in the field of the nuclei (number density N, charge Z), we get the following expression for the low frequency radiation spectrum (for a non-relativistic electron):

\[
I(\omega) = \frac{Z^2 e^6 N}{12 \pi^3 \varepsilon_0^3 c^3 m_e^2} \frac{1}{v} \ln \frac{b_{\text{max}}}{b_{\text{min}}}
\]

\(b_{\text{min}}\) is given as in our ionization calculation (\(\hbar / 2m_e v\)).

For a given frequency, \(b_{\text{max}}\) is given by the condition for the cutoff frequency.

Thus we get:

\[
\frac{b_{\text{max}}}{b_{\text{min}}} = \frac{2 m_e v^2}{\hbar \omega}
\]

and the energy loss rate is given by:

\[
- \left( \frac{dE}{dt} \right)_{\text{brems}} = \int_0^{\omega_{\text{max}}} I(\omega) \, d\omega
\]

We have to integrate to a maximum frequency \(\omega_{\text{max}}\), which corresponds to a maximum energy transfer.
Bremsstrahlung - non-relativistic limit (3)

maximum energy transfer if:
photon energy = kinetic energy of electron

\[ \hbar \omega_{\text{max}} = \frac{1}{2} m_e v^2 \rightarrow \omega_{\text{max}} = \frac{m_e v^2}{2 \hbar} \]

Thus we find:

\[-\left( \frac{\text{d}E}{\text{d}t} \right)_{\text{brems}} = \int_0^{\omega_{\text{max}}} \frac{Z^2 e^6 N}{12 \pi^3 \varepsilon_0^3 c^3 m_e^2} \frac{1}{v} \ln \left( \frac{2 m_e v^2}{\hbar \omega} \right) \, d\omega = \]

\[= (\text{const.}) Z^2 N v \]

- total energy loss rate proportional \( v \) (velocity of the electron), i.e. proportional to \( \sqrt{E_{\text{kin}}} \).
- in practical applications (i.e. X-ray emission from solar flares) one has to integrate the energy loss over a distribution of electron velocities.
- \( \text{d}E/\text{d}t \sim 1/m^2 \) => bremsstrahlung mostly negligible for heavier particles.
An example: spectrum of the Perseus galaxy cluster

This X-ray spectrum of the Perseus Cluster was obtained with Ariel V. An emission feature is visible at about 7keV, due to Fe XXV and Fe XXVI. This feature provides strong evidence for the presence of hot plasma in the cluster. The overall spectrum is well described by bremsstrahlung emitted from an adiabatic hydrostatic atmosphere of hot gas in the gravitational potential well of the cluster.

(Image credit: R.J. Mitchell, et al., MSSL)
Another example: the sun in X-rays

Soft X-ray thermal bremsstrahlung coronal image of the sun, taken by the Yohkoh Solar Observatory on February 1st, 1992. It highlights the magnetic field loop structure that forms the precursor environment to solar flare activity.

http://spacibm.rice.edu/~baring/phys541
A full relativistic quantum treatment results in the bremsstrahlung formula derived by Bethe and Heitler:

\[- \left( \frac{dE}{dt} \right) = \frac{Z(Z+1.3)e^6N}{16\pi^3\varepsilon_03m_e2c^4\hbar} \cdot E \cdot \left[ \ln \left( \frac{183}{Z^{1/3}} \right) + \frac{1}{8} \right] \]

- \((Z+1.3)\) accounts for interactions between the high energy electron and the bound electrons
- \(Z^{-1/3}\) accounts for screening of nuclei by bound electrons (scales with the radius of the atom).
- the energy loss rate in the relativistic case is proportional to the kinetic energy of the h.e. electron. (different from non-relat. case!)

**Radiation Length**

for highly relativistic electrons: \( v \approx c \)

\[- \frac{dE}{dt} = - \frac{dE}{dx} \frac{1}{c} \rightarrow - \frac{dE}{dx} \propto E \]

\[- \frac{dE}{dx} \equiv \frac{E}{X_0} \rightarrow E(x) = E_0 \exp\left( - \frac{x}{X_0} \right) \]
Bremsstrahlung - relativistic limit & radiation length (2)

The constant $X_0$ is called «radiation length». When traversing one radiation length, the electron loses a fraction of $(1 - 1/e)$ of its energy. It is often described as a grammage $\xi_0 = \rho \ X_0$.

$$- \frac{dE}{d\xi} = - \frac{dE}{dt} \frac{1}{\rho \ c} = \frac{E}{\rho X_0} = \frac{E}{\xi_0} \rightarrow E(\xi) = E_0 \exp(-\frac{\xi}{\xi_0})$$

This is also called the stopping power of a material. We can find a numerical expression by replacing $\rho$ with $N * \frac{M_{\text{atom}}}{N_A}$ and by inserting the Bethe-Heitler result for $- \frac{dE}{dt}$:

$$\xi_0 = \frac{7160 M_{\text{atom}}}{Z(Z+1.3)\left[\ln(183 Z^{-\frac{1}{3}})+\frac{1}{8}\right]} \text{ kg m}^{-2}$$

radiation lengths for different materials:

- hydrogen $\xi_0 = 580 \text{ kg/m}^2 \quad X_0 = 6.7 \text{ km}$
- air $\xi_0 = 365 \text{ kg/m}^2 \quad X_0 = 280 \text{ m}$
- lead $\xi_0 = 58 \text{ kg/m}^2 \quad X_0 = 5.6 \text{ mm}$
For $E_{\text{kin}}$ below $\sim 1$ MeV the electron becomes non-relativistic. For lower energies, ionization losses are the dominant loss mechanism. For greater energies, relativistic bremsstrahlung losses become dominant.

The critical energy is defined as the energy at which bremsstrahlung losses and ionization losses become equal.

Critical energy for electrons in:

- hydrogen: 340 MeV
- air: 83 MeV
- lead: 6.9 MeV

For protons and other heavy particles, the critical energy is very high and bremsstrahlung does not play an important role compared to ionization losses.
Synchrotron Radiation

- Not an interaction of particles with matter, but with a magnetic field.

- Ultra-relativistic electrons (and other charged particles) are accelerated in a magnetic field and give off electromagnetic radiation. The physical process is thus somewhat similar to bremsstrahlung.

- In Astrophysics: explains X-ray (and gamma-ray ?) emission of AGN; radiation also at radio and optical frequencies.

- In Particle Physics: synchrotron radiation is a very important energy loss mechanism for synchrotron and cyclotron accelerators (esp. for electrons).

- Due to the highly relativistic energy of the electron, synchrotron emission is strongly forward beamed, i.e. emitted in a cone in the direction of the particle's motion. It is also polarised.

- At lower (non-relativistic) energies, the emission is called "cyclotron radiation".
Synchrotron Radiation - some formulas (1)

We begin again with the Larmor formula for accelerated charges:

\[
P = - \left( \frac{dE}{dt} \right)_{\text{rad}} = \frac{q^2 |\mathbf{r}|^2}{6\pi \epsilon_0 c^3}
\]

We need an expression for the acceleration of a highly relativistic particle (perpendicular to the direction of motion!) in the particle's rest frame:

\[
|\ddot{\mathbf{r}}| = \frac{1}{m} \frac{dp}{d\tau} = \frac{1}{m} \frac{d(\gamma m v)}{d(t/\gamma)} \approx \gamma^2 \frac{d\gamma}{dt} = \gamma^2 \frac{v^2}{r}
\]

Here we have made the assumption that the magnitude of the velocity does not change in one orbit, only its direction (centripetal acceleration). We also assume that the velocity is perpendicular to the magnetic field lines (pitch angle = 90°).

We thus get the following expression for the radiated power:

\[
P = - \left( \frac{dE}{dt} \right)_{\text{rad}} = \frac{q^2 \gamma^4 v^4}{6\pi \epsilon_0 c^3 r^2} \approx \frac{q^2 c \gamma^4}{6\pi \epsilon_0 r^2}
\]

where \( r \), the radius of gyration, depends on the magnetic field, particle momentum and charge:

\[
r = \frac{\gamma m v}{q B}
\]
Synchrotron Radiation - some formulas (2)

- The synchrotron loss rate scales with the 4th power of the particle energy $\gamma mc^2$.

- The synchrotron loss rate is inversely proportional to the square of the radius of the particle's orbit. (Larger radius in accelerators -> less synchrotron losses!)

Finally, let's calculate the energy loss per orbit:

$$\Delta E = P \cdot \left( \frac{2\pi r}{v} \right) = \frac{q^2 \gamma^4 v^3}{3 \varepsilon_0 c^3 r} = \frac{1}{3 \varepsilon_0} \frac{q^2 \gamma^4 \beta^3}{r}$$

How do the synchrotron loss rates for an electron and a proton of the same energy compare?

$$E_e = \gamma_e mc^2 = E_p = \gamma_p m_p c^2 \implies \gamma_e / \gamma_p = m_p / m_e \sim 1836$$

$$\frac{P_p}{P_e} = \frac{\gamma_p^4}{\gamma_e^4} = \frac{m_e^4}{m_p^4} \approx \frac{1}{1836^4} \approx 9 \cdot 10^{-14}$$

Thus, the synchrotron loss rate for protons is often negligible.
Example: Synchrotron Emission from M87 (jet)

source: NASA

HST optical

VLA Radio
An example: synchrotron emission model for AGN

modelling of the X-ray and gamma-ray emission from blazar PKS 2155-304 assuming:
- synchrotron emission from electrons
  -> X-rays
- synchrotron emission from protons
  -> gamma-rays

simulations by E. Ferrière, 2009
Cherenkov radiation

- Emission of Cherenkov photons takes place when a charged particle moves through a medium (an insulator!) at a constant velocity $v$ which is greater than the velocity of light in that medium ($c/n$).
- The charged particles polarize the dielectric medium, which then returns into its ground state, emitting radiation.
- The wavefronts of the emission from the superluminal particle add up coherently in a direction with $\cos \phi = c / (n \cdot v)$ against the direction of motion of the particle. Cherenkov light is thus emitted in a characteristic cone around the particle's direction of motion. At sub-luminal velocities, the emitted photons interfere destructively.
- Cherenkov radiation is widely used in Cherenkov and threshold detectors to measure the velocity and direction of relativistic particles in Astrophysics and Particle Physics.
A supersonic jet aircraft:
The pressure waves caused by the aircraft superpose in a shock front that travels at the speed of sound. This shock front is audible as a single "sonic boom" on the ground. This is the same principle as the light wavefront in the case of Cherenkov light emission.

In the photograph, the conical shock front is visible as the aircraft passes the sound barrier because water vapour condenses in the zone where the air pressure changes.

simulation: http://www.shef.ac.uk/physics/teaching/phy311/animations.html
Cherenkov radiation - some formulas (1)

Derivation of Cherenkov radiation is quite elaborate. We will just give a summary of the results here.

1. **Condition for coherent radiation**

\[ v > \frac{c}{n} \quad ; \quad n > 1 \]

\[ \cos \theta = \frac{c}{n \, v} = (\beta n)^{-1} \]

- \( n \) is the refractive index of the medium. \( n = \sqrt{\varepsilon} \)
- \( n \) is a function of wavelength: \( n(\omega) \)

- There is a **minimum velocity** for the Cherenkov effect, which depends on the medium: \( \beta = 1 / n \)
- For an ultra-relativistic particle (\( \beta \sim 1 \)), there is a **maximum emission angle** \( \cos \theta = 1 / n \)
- There are two further conditions to be fulfilled to achieve coherence:

First, the **length of the track** of the particle in the medium should be **large compared to the wavelength** of the radiation in question, otherwise diffraction effects will become dominant.

Second, the **velocity of the particle must be nearly constant** during its passage through the medium, i.e. the time difference for traversing two successive distances of order of the wavelength should be small.
Cherenkov radiation - some formulas (2)

2. Energy loss rate per unit path length / unit time

\[
\frac{dU(\omega)}{dx} = \frac{\omega z^2 e^2}{4 \pi \varepsilon_0 c^3} \left(1 - \frac{c^2}{n^2 v^2}\right)
\]

Emission is proportional \( 1 / \lambda \)

\[
I(\omega) = \frac{dU(\omega)}{dt} = \frac{dU(\omega)}{dx} \frac{dx}{dt} = \frac{\omega z^2 e^2 v}{4 \pi \varepsilon_0 c^3} \left(1 - \frac{c^2}{n^2 v^2}\right)
\]

- By measuring the total Cherenkov light signal, one can get information on the **charge** \( z \) and **velocity** \( v \) of the high energy particle in the medium.
- The directed emission allows determination of the particle's **direction**.
- The radiation occurs in the **UV, (near) visible and radio regions** of the spectrum, for which \( n > 1 \). In this wavelength range, the radiation increases with the frequency (\( \rightarrow \) largest signal in UV !). In the X-ray waveband, \( n \) is always \( < 1 \) and radiation is forbidden.
- The **energy loss due to ionisation or excitation is two to three orders of magnitude higher than the energy lost in radiating Cherenkov light**, in the energy range where photomultipliers can be used (a few eV, or about 400 nm wavelength).
some Cherenkov detector types

- **Threshold Detectors**
  - We have seen that we have a Cherenkov signal only for \( v > c/n \). Choosing a medium with a certain refractive index \( n \) defines thus a velocity threshold for particle detection.
  - for \( v \geq 0.6 \, c \): solid dielectric materials with \( n \approx 1.5 \) (e.g. lucite, plexiglass...)
  - for ultra-relativistic particles: gas Cherenkov detectors with \( n \geq 1 \)
  - The total emitted light gives information on particle charge and velocity. With a series of different threshold detectors, one can determine both charge and velocity of the particle.
  - The light signal is very weak and has to be amplified with photomultipliers.

- **Ring Imaging Cherenkov Detectors** (used e.g. by DELPHI at LEP):
  measures the velocity of the high energy particle by determining the angle of the Cherenkov cone.

- **Water Cherenkov Detectors** (used e.g. by the Pierre Auger Observatory):
  large water tanks equipped with photomultiplier tubes record the Cherenkov light emitted from traversing charged particles. The recorded light pulses are used to gain information of the extensive air showers that are caused by ultra-high energy cosmic rays in the atmosphere (see Chapter 6).

- **Imaging Air Cherenkov Telescopes** (e.g. Magic, HESS...):
  record an image of the Cherenkov emission of extensive air showers from gamma-rays (see Chapter 5).
Transition radiation

- Transition radiation is known as the « sub-threshold » emission of Cherenkov light of charged particles, even with velocity $< \frac{c}{n}$.

- **Transition radiation occurs when particles traverse a thin segment of material.** The transition in the index of refraction ($n$) between the vacuum and the material leads to emission of Cherenkov light in the X-ray range. Roughly speaking, the electric displacement field of the particle ($\vec{D} = \varepsilon \vec{E}$) changes when crossing to another medium; the energy difference is emitted in X-rays. Diffraction effects cause some non-destructive interference even at energies below $\frac{c}{n}$.

- Transition radiation is used in detectors for the velocity measurement and particle identification at **highly relativistic energies.** (e.g. cosmic ray detectors for muons at TeV energies)
Transition radiation – some formulas

Total emitted radiation when crossing one boundary:

\[ S = \frac{1}{3} \alpha z^2 \hbar \omega_p \gamma \quad ; \quad \alpha = \frac{1}{137} \quad ; \quad \omega_p = \sqrt{\frac{4\pi N_e e^2}{m_e}} \]

- The « plasma frequency» \( \omega_p \) is a characteristic of the material. \( N_e \) is the electron density in the material.
- The emitted radiation is proportional to \( \gamma \) (not \( v \) !), i.e. to the particle's energy.
- Radiation in X-ray regime. The radiation yield drops sharply for \( \omega > \gamma \omega_p \).
- Emission is mostly directed forward, peaking at an angle of the order of \( 1 / \gamma \) relative to the particle's path.

Minimum foil thickness:

The photon yield at each crossing is very small, but can be increased by multiple transitions. Thin foils of appropriate materials are used (e.g. Li). The foil thickness is an important parameter. It should not be too thick, to minimise absorption of the emitted X-rays, but it has to have a minimum thickness to allow constructive interference. This minimum is given by: \( d \geq \frac{\gamma c}{\omega_p} \)
Transition Radiation Detectors

- A practical transition radiation detector consists of two parts:
  - a **stack of thin foils of radiator** (to produce the TR X-rays)
  - followed by a special **multiwire proportional chamber or calorimeter** to detect them *(see Chapter 3)*

- "The radiator is made of a low atomic number material, to minimise absorption of the X-ray photons. Lithium foils in a helium atmosphere is ideal, for example 400 foils of thickness 40 microns, spaced 160 microns apart by corrugations on alternate foils.* *(from Dr. C. N. Booth)*

- TRDs are used in particle accelerator experiments (e.g. ATLAS) and cosmic ray experiments (e.g. AMS)

*taken from the website of Dr. C. N. Booth (http://www.shef.ac.uk/physics/teaching/phy311/coherent.html)*
Electron-Positron Annihilation

- The most extreme form of energy loss is particle-antiparticle annihilation. The particle and its antiparticle collide and their freed energies are used for the production of high energy photons.
- This is interesting in the case of electrons and positrons, since positrons are generated in astrophysical environments (decay of $\pi^+$, $\beta^+$ decay of radioactive elements, pair production...)
- The inverse process, particle-antiparticle pair production, will be discussed in the next chapter.
- The existence of positrons in the Galactic Center could be proven by observation of the electron-positron annihilation line at 0.511 MeV in $\gamma$-ray astronomy.
Electron-Positron Annihilation - some background (1)

Electron-positron annihilation can proceed in two ways:

1. $e^+ e^- \rightarrow 2 \gamma$

When the electron and positron are at rest when interacting, both photons have energy 0.511 MeV. When they interact «in flight», there is a range of possible photon energies.

2. positronium atoms

If the velocity of the positron and electron is small, positronium atoms, i.e. bound states of an electron and a positron can form.

25% of the p.a. form in the singlet $^1S_0$ state ("para-positronium"), 75% in the triplet $^3S_1$ state ("ortho-positronium").

The **singlet state decays into two $\gamma$-rays**, both with energy 0.511 MeV (lifetime: $1.25 \times 10^{-10}$ s).

The **triplet state decays into three $\gamma$-rays**, with a range of energies, with maximum energy 0.511 MeV (lifetime: $1.5 \times 10^{-7}$ s).
Electron-Positron Annihilation - some background (2)

The formation of positronium atoms is only possible in regions that are not too dense and not too hot. The characteristic shape of the 0.511 MeV spectral line is thus a diagnostic tool in γ-ray astronomy.

Cross-section for $e^+ e^-$ annihilation

- **Extreme relativistic limit:**
  \[ \sigma = \frac{\pi R_e^2}{\gamma} \left[ \ln 2 \gamma - 1 \right] \]
  \[ r_e = \frac{\alpha \hbar}{m_e c} \]

- **Thermal electrons and positrons:**
  \(~25\,\text{meV}, E_{\text{kin}} \sim kT\)
  \[ \sigma \approx \frac{\pi R_e^2}{\beta} \]

Spectrum of the $e^+ e^-$ annihilation radiation detected by SPI (Integral) towards the GC region after 3.5 million seconds exposure. The red line shows the positron annihilation line, the blue line shows the continuum spectrum associated with the three photon ortho-positronium decay.
Nuclear Interactions

- Inelastic collisions of high energy particles with nuclei lead to nuclear interactions. We will have a quick look at nuclear spallation and emission processes.

- Nuclear interactions of high energy particles are very important in the study of extensive air showers, where high energy cosmic rays collide with nuclei in the atmosphere.

- Production of neutrinos by nuclear decay is another important aspect for astroparticle physics. We will save this for a later chapter on neutrino experiments (see chapter 7).
A general (simple) picture of nuclear spallation (1)

- The strong interaction force is a short range force. We need thus a direct collision of the high energy particle with the nucleus to have a nuclear interaction.

- Thus, we can describe the cross-section of the nucleus roughly with its geometrical cross-section. Its radius is given as \( R \sim 1.2 \times 10^{-15} A^{1/3} \text{ m} \), where \( A \) is the mass number.

What happens when a high energy proton interacts with a nucleus?

- The effective size of a high energy proton is small. We can imagine it interacting with individual nucleons inside the nucleus.

- The collision produces many pions, some strange particles (e.g. Kaons) and eventually antinucleons. The hit nucleons can be expelled as well.

- Nucleons and pions have high energies. They can interact with other nucleons in the core. They leave the nucleus mostly in the forward direction of the h.e. particle.
Spallation

Intra-nuclear cascade

Impinging fast particles

Target nuclei

Inter-nuclear cascade

Cascade particle

~ 1 Giga electronvolt

Highly excited nucleus

Evaporation

Proton

Neutron

taken from:
http://www.kawo1.rwth-aachen.de/~amarock/ess/pages/spallation.htm
A general (simple) picture of nuclear spallation (2)

- After the interaction, the nucleus is usually left in a highly excited state and with a deficit of nucleons. Spallation fragments evaporate from the nucleus isotropically. Neutrons can be emitted as well.

- If the high energy particle is not a proton, but a nucleus itself, both nuclei (target and projectile) will undergo spallation.

- High energy photons can also cause spallation of nuclei.

Some useful approximate numbers:

The mean free path of a high energy proton in the atmosphere is about 800 kg / m$^2$. (Depth of the atmosphere is about 10 000 kg / m$^2$.) Thus, the probability of having a nuclear interaction occur for a pathlength of x kg / m$^2$ is $1 - \exp(-x / (800 \text{ kg/m}^2))$.

For high energy protons with energy $> 1$ GeV in collision with air nuclei, roughly $2 \times (E/\text{GeV})^{1/4}$ new high energy, charged particles are generated.
Nuclear Interactions - Nuclear emission lines (1)

Nuclear processes can produce $\gamma$-ray lines in the spectra of astrophysical sources. Two types of nuclear processes are possible.

1. Nuclear photo-excitation

Nuclei can be excited to energy levels above the ground state by collisions with cosmic rays (peripheral collision) and gamma rays. In the subsequent decay to the ground state, the nucleus emits the energy difference in the form of $\gamma$-rays. (~ similar to photo-excitation of atoms)

The cross-section for excitation of the nucleus attains a maximum value for energies that correspond to the difference in energy levels -> spectral lines.

These interactions take place in the diffusive interstellar gas or in interstellar dust grains.

Figure 5.7. The predicted $\gamma$-ray spectrum resulting from low energy cosmic ray interactions with the interstellar gas in the general direction of the Galactic Centre. (From R. Ramaty and R. E. Lingenfelter (1979). Nature, 278, 127.)

Broad lines from collisions in the gas phase, narrow lines produced from dust grains. e+ e- annihilation line at 0.511 MeV.
2. Decay of radioactive isotopes

Supernova explosions generate stable and unstable nuclei. The radioactive decay of the unstable nuclei is another source of γ-ray emission.

The observation of spectral lines associated with certain radioactive isotopes (\(^{56}\text{Ni},^{56}\text{Co},^{56}\text{Fe} \ldots\)) gives insight into the mechanism of supernova explosions.

Apart from the short-lived isotopes one observes after the explosion, there are also longer-lived isotopes (\(^{60}\text{Co},^{26}\text{Al} \ldots\)) that are generated in supernovae and ejected into the interstellar medium. These make up a diffuse flux of γ-ray line emission.