Filamentation instability of long Alfvén waves in warm collisionless plasmas

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A criterion for filamentation instability of a circularly polarized Alfvén wave train propagating along a strong ambient field in a collisionless plasma is obtained from a transverse modulational analysis of a generalized kinetic derivative nonlinear Schrödinger equation. This equation, which retains Landau damping, results from the Vlasov–Maxwell equations via a long-wave reductive perturbative expansion. For bi-Maxwellian equilibrium distribution functions, it is shown that a moderate anisotropy of the electron temperature and/or a moderate ratio of the electron to ion temperatures can significantly broaden the existence range of this instability, making its occurrence possible in small-β plasmas. [DOI: 10.1063/1.1611487]

I. INTRODUCTION

Dispersive Alfvén wave turbulence is commonly observed in space plasmas, especially in the solar wind,[1] the magnetosheath,[2,3] and auroral regions.[4,5] In places where the plasma is transversally structured, kinetic Alfvén waves can develop,[6] which contribute to plasma energization and particle acceleration. It is thus of interest to look for self-consistent theories to explain the formation of transverse density inhomogeneities in an initially homogeneous plasma. A mechanism by which linearly polarized obliquely propagating Alfvén waves generate transverse gradients through the formation of pressure balanced structures is discussed by Vasquez and Hollweg.[7] The present paper focuses on a process by which a circularly polarized parallel propagating dispersive Alfvén wave train undergoes a transverse modulational instability that leads to wave energy concentration in field-aligned magnetic filaments. This phenomenon, called Alfvén wave filamentation, could become relevant in the interpretation of recent CLUSTER observations of current tubes in magnetosheath regions close to the bow shock where quasimonochromatic Alfvén ion-cyclotron waves were also identified.[8]

Alfvén wave filamentation was extensively studied in the context of Hall-magnetohydrodynamics for a polytropic gas, a description adapted to collisional media but that also extends to collisionless plasmas with a small β, provided anisotropy pressure effects are not important. In the latter case, filamentation instability is convective, in the sense that a time-stationary wave collapse develops along the direction of propagation.[9] Direct numerical integration of the primitive equations being particularly challenging in this regime, simulations were performed for $\beta>1$, a case where the instability is absolute and develops in time. As usual for weakly nonlinear dispersive waves, the transverse dynamics (either convective or absolute) is governed to leading order by the two-dimensional nonlinear Schrödinger equation for the amplitude of the pump. Quantitative comparisons between the theoretical predictions and the results of direct numerical simulations demonstrate the accuracy of this description.[10]

The question then arises of the influence of the kinetic effects on Alfvén wave filamentation. Retaining these effects is usually a main difficulty in the description of the large-scale dynamics of a collisionless plasma, numerical integration of the Vlasov–Maxwell equations in three space dimensions being much beyond the capabilities of the present day computers. It turns out that kinetic effects can be incorporated in a fluid description when considering Alfvén waves with typical wavelengths much longer than the ion Larmor radius (and/or the ion inertial length) and amplitudes small enough for dispersive and nonlinear effects to be comparable. In this asymptotic regime, some decoupling takes place and the dynamics greatly simplifies.

When a long-wave reductive perturbative expansion is performed on the Hall-magnetohydrodynamic equations (that ignore kinetic effects), a multidimensional derivative nonlinear Schrödinger equation (DNLS) is obtained for the transverse magnetic field. When dealing with wave trains, the coupling of the Alfvén wave with magnetosonic waves at much larger scale is relevant. In a reductive perturbative expansion that retains only one longitudinal scale, the magnetosonic waves enter in the form of average values along the direction of propagation.[11] Absolute filamentation, which in this long-wave limit occurs for $\beta>1$, is reproduced by such a generalized DNLS equation.[12]

A similar perturbative analysis can also be performed on the Vlasov–Maxwell equations, leading to the kinetic derivative nonlinear Schrödinger equation (KDNLS). The formulation suitable for Alfvén wave trains was derived in Passot and Sulem[14] by extending a previous work by Rogister,[15] limited to localized pulses. The resulting equation turns out to be a slight modification of the DNLS equation, that includes the Landau damping. This asymptotic model is the starting point of the present paper that is organized as follows. Section II recalls the KDNLS system for Alfvén wave trains. In Sec. III, a modulational analysis for a monochromatic Alfvén wave subject to transverse perturbations is performed, resulting in a two-dimensional nonlinear Schrödinger equation with linear damping. Section IV specifies the criterion for filamentation instability in the case of bi-Maxwellian equilibrium distribution functions. Section V analyzes this criterion in a few typical regimes. The important role played by the electron temperature anisotropy and by the ratio of electron to proton temperatures is pointed out. The influence of a second ion species, such as He, is also considered. Section VI is a brief conclusion that also suggests further developments, in particular to situations where, as in the solar wind or the magnetosheath, distribution functions may display significant deviations from bi-Maxwellians.

II. LOW-FREQUENCY ALFVÉN WAVE TRAINS

As already mentioned, the dynamics of long-Alfvén waves in a collisionless plasma is isolated by performing a reductive perturbative expansion on the Vlasov–Maxwell equations.[14] This yields a closed system for the leading order transverse magnetic field $\epsilon \mathbf{b}_\perp^{(0)}$ and the fluctuating longitudinal magnetic disturbance $\epsilon^2 b_\parallel^{(1)}$ (two quantities related by the magnetic field divergenceless condition), in terms of the stretched longitudinal variable $\xi=\epsilon^2(x-\lambda t)$, of the transverse ones $\eta=\epsilon^3 y$ and $\zeta=\epsilon^3 z$ (relatively to the ambient field), and of the slow time $\tau=\epsilon^4 t$. Using the complex notations $b_\perp^{(0)}=b_y^{(0)}+i b_z^{(0)}$ and $\partial_\perp=\partial_\eta+i\partial_\zeta$, this system reads

$$(\partial_\tau+\langle U\rangle\partial_\xi)b_\perp^{(0)}+\partial_\xi\left(\frac{\tilde{P}b_\perp^{(0)}}{2\lambda\rho^{(0)}}\right)-\frac{B_0}{2\lambda\rho^{(0)}}\partial_\perp\tilde{P}+i\delta\partial_{\xi\xi}b_\perp^{(0)}=0, \tag{1}$$

$$\rho^{(0)}\partial_\tau\langle U\rangle=C_1\left(\Re\left\{\partial_\perp^*\left\langle\tilde{P}\frac{b_\perp^{(0)}}{B_0}\right\rangle\right\}-\langle\tilde{A}\mathcal{K}\partial_\xi\tilde{A}\rangle\right)-C_2\langle\langle\tilde{A}\mathcal{K}\partial_\xi\tilde{A}\rangle\rangle, \tag{2}$$

$$\partial_\xi\tilde{b}_\parallel^{(1)}+\Re\{\partial_\perp^* b_\perp^{(0)}\}=0. \tag{3}$$

The notation $\Re\{\partial_\perp^*\phi\}$, where $\phi=\phi_R+i\phi_I$ is a complex field, holds for the (transverse) divergence $\partial_\eta\phi_R+\partial_\zeta\phi_I$. The quantity $\tilde{P}=((B_0^2/4\pi)+2p_\perp^{(0)}+\mathcal{K})\tilde{A}$ refers to the leading order perturbation of the perpendicular total pressure (magnetic and thermodynamic). It is expressed in terms of the fluctuations of $\tilde{A}$ of the normalized magnetic field amplitude perturbation $A=(b_\parallel^{(1)}/B_0)+(|b_\perp^{(0)}|^2/2B_0^2)$ about its longitudinal mean value $\langle A\rangle$ (brackets $\langle\cdot\rangle$ denote averaging over the ξ variable, while double brackets $\langle\langle\cdot\rangle\rangle$ hold for averaging over the full spatial domain). The above system also involves the kinetic operator $\mathcal{K}=\mathcal{N}-\mathcal{M}^2\mathcal{L}^{-1}$, where $\mathcal{L}=\Sigma_r\mathcal{L}_r$, $\mathcal{M}=\Sigma_r\mathcal{M}_r$, and $\mathcal{N}=\Sigma_r\mathcal{N}_r$ originate from the resonance of the wave with the particles of various species r. For each particle species of mass m_r, charge q_r and number density n_r, one has

$$\begin{aligned}\mathcal{L}_r&=2\pi\frac{q_r^2 n_r}{m_r}\int_0^\infty d\left(\frac{v_\perp^2}{2}\right)\mathcal{G}_r,\\ \mathcal{M}_r&=2\pi q_r n_r\int_0^\infty d\left(\frac{v_\perp^2}{2}\right)\frac{v_\perp^2}{2}\mathcal{G}_r,\\ \mathcal{N}_r&=2\pi m_r n_r\int_0^\infty d\left(\frac{v_\perp^2}{2}\right)\frac{v_\perp^4}{4}\mathcal{G}_r,\end{aligned} \tag{4}$$

where the operator

$$\mathcal{G}_r=P\int\frac{1}{v_\parallel-\lambda}\frac{\partial F_r^{(0)}}{\partial v_\parallel}dv_\parallel+\pi\left.\frac{\partial F_r^{(0)}}{\partial v_\parallel}\right|_{v_\parallel=\lambda}\mathcal{H}_\xi \tag{5}$$

includes a Hilbert transform $\mathcal{H}_\xi$ with respect to the longitudinal variable ξ. Furthermore, the dispersion coefficient δ that arise Eq. (1) scales like the inverse of the ion gyrofrequency (its precise form is not needed in the present analysis), while the Alfvén wave propagation velocity λ is defined by $\lambda^2\rho^{(0)}=((|B_0|^2/4\pi)+p_\perp^{(0)}-p_\parallel^{(0)})$. The density $\rho^{(0)}$ and the transverse and parallel pressure components $p_\perp^{(0)}$ and $p_\parallel^{(0)}$ at equilibrium are constructed from the velocity distribution function $F^{(0)}(v_\perp,v_\parallel)$ assumed rotationally invariant about the direction of the ambient field $B_0\hat{x}$ and symmetric relatively to forward and backward velocities along this direction, thus excluding the presence of equilibrium drifts.[16,17]

When restricted to spatially localized solutions, the longitudinally averaged fields are zero, and one recovers the multidimensional DNLS equation derived by Rogister[15] and Mjølhus and Wyller.[18] In contrast, when dealing with wave trains, the system also includes the mean field $\langle U\rangle$ that results from the combination

$$\langle U\rangle=\langle u_\parallel^{(1)}\rangle+\frac{\lambda}{2B_0}\langle b_\parallel^{(1)}\rangle+\frac{1}{\lambda\rho^{(0)}}(p_\parallel^{(0)}-p_\perp^{(0)})\langle A\rangle+\frac{1}{2\lambda\rho^{(0)}}(\langle p_\perp^{(1)}\rangle-\langle p_\parallel^{(1)}\rangle) \tag{6}$$

of various longitudinally averaged quantities, namely the hydrodynamic velocity, the longitudinal induced magnetic field, the normalized magnetic perturbation, and longitudinal and transverse pressure perturbations. Furthermore, the constants

$$\begin{aligned}C_1&=\frac{1}{2+\beta_\perp-\beta_\parallel}\left(\frac{12+18\beta_\perp+5\beta_\perp^2}{8(1+\beta_\perp)}\right),\\ C_2&=\frac{1}{2+\beta_\perp-\beta_\parallel}\left(\frac{(2+\beta_\perp)^2}{8(1+\beta_\perp)}\right)\end{aligned} \tag{7}$$

involve the ratios $\beta_\parallel=8\pi p_\parallel^{(0)}/B_0^2$ and $\beta_\perp=8\pi p_\perp^{(0)}/B_0^2$ of the magnetic and parallel or transverse pressure at equilibrium, respectively.

III. FILAMENTATION INSTABILITY

Equations (1)–(3) are valid for (normalized) wave amplitudes $b_\perp^{(0)}/B_0$ that are, at the maximum, of the order of the square root of the typical ratio of the ion inertial length to the typical wavelength. If the amplitude is smaller, the dynamics on the time scale τ reduces to that of linear waves with a dispersion relation $\omega=\delta k^2$. A richer dynamics can nevertheless take place on a longer time scale, on which the large-scale modulation of a (circularly polarized) monochromatic Alfvén wave can, in some instances, lead to instabilities and to the onset of a nonlinear regime. In order to describe the transverse instability of such a wave, one introduces the slow transverse variables $Y=\epsilon\eta$ and $Z=\epsilon\zeta$ together with a slow time $T=\epsilon^2\tau$, associated with this effect, and write that, to leading order, $b_\perp^{(0)}=\epsilon\psi(Y,Z,T)e^{i(k\xi-\omega\tau)}$, with the dispersion relation $\omega=\delta k^2$. As demonstrated in the context of the Hall-magnetohydrodynamics, the succession of a reductive perturbative expansion and of a modulational analysis is equivalent to first performing a modulational analysis on the primitive equations and then take the long wave limit $k\to 0$ in the resulting envelope equations.[9,12] Taking such a limit implies the amplitude of the pump to be reduced accordingly, in order to keep the nonlinear effects negligible on the time scale of the wave period.

It follows from the above asymptotics that, still to leading order, $b_\parallel^{(1)}=\epsilon^2\Re\{i/k\partial_\perp^*\psi e^{i(k\xi-\omega\tau)}\}$ and $\widetilde{P}=2\epsilon^2\Re\{(P_1+iP_2)i\partial_\perp^*\psi e^{i(k\xi-\omega\tau)}\}$ with $P_1=(1/2kB_0)(B_0^2/4\pi+2p_\perp^{(0)}+K_1)$ and $P_2=K_2/2|k|B_0$, where we wrote $\mathcal{K}=K_1+K_2\mathcal{H}_\xi$. It follows that

$$\partial_\perp\widetilde{P}=(P_1+iP_2)i\Delta_\perp\psi e^{i(k\xi-\omega\tau)}+(\cdots)e^{-i(k\xi-\omega\tau)}, \tag{8}$$

where the dots hold for a slowly varying quantity that need not to be explicited. As a consequence, the modulation of the pump amplitude is governed by the nonlinear Schrödinger equation,

$$i\partial_T\psi+\frac{B_0}{2\lambda\rho^{(0)}}(P_1+iP_2)\Delta_\perp\psi-k\langle U\rangle\psi=0, \tag{9}$$

that includes a linear diffusive term associated with the Landau damping, in addition to the diffraction and to the potential $\langle U\rangle$ to be determined in terms of ψ, by means of Eq. (2).

Note that in the case of transverse modulation of a long pump wave, diffraction and damping originate from the transverse pressure gradient, and that the nonlinear coupling are only due to the mean fields that it is essential to retain.[13] In order to evaluate $\langle U\rangle$, one computes $\langle\widetilde{P}b_\perp^{(0)}\rangle$ and

$$\Re\{\partial_\perp^*\langle\widetilde{P}b_\perp^{(0)}\rangle\}=-P_2|\partial_\perp^*\psi|^2+\Re\{i(P_1+iP_2)\psi^*\Delta_\perp\psi\}. \tag{10}$$

Since

$$\partial_T|\psi|^2-\frac{B_0}{\lambda\rho^{(0)}}\Re\{i(P_1+iP_2)\psi^*\Delta_\perp\psi\}=0, \tag{11}$$

one gets

$$\Re\{\partial_\perp^*\langle\widetilde{P}b_\perp^{(0)}\rangle\}=-P_2|\partial_\perp^*\psi|^2+\frac{\lambda\rho^{(0)}}{B_0}\partial_T|\psi|^2. \tag{12}$$

In addition

$$\widetilde{A}=\frac{b_\parallel}{B_0}=\Re\left\{\frac{i}{kB_0}\partial_\perp^*\psi e^{i(k\xi-\omega\tau)}\right\} \tag{13}$$

and

$$\langle\widetilde{A}K\partial_\xi\widetilde{A}\rangle=-\frac{1}{2B_0^2|k|}K_2\langle|\partial_\perp\psi^*|^2\rangle. \tag{14}$$

Furthermore,

$$\langle\langle|\partial_\perp^*\psi|^2\rangle\rangle=-\langle\langle\Re\{\psi^*\Delta\psi\}\rangle\rangle=-\frac{\lambda\rho^{(0)}}{B_0}\partial_T\langle\langle|\psi|^2\rangle\rangle. \tag{15}$$

One has

$$\partial_T\langle U\rangle=\frac{\lambda}{B_0^2}\partial_T(C_1|\psi|^2+C_2\langle\langle|\psi|^2\rangle\rangle). \tag{16}$$

It follows that

$$\langle U\rangle=\frac{\lambda}{B_0^2}(C_1|\psi|^2+C_2\langle\langle|\psi|^2\rangle\rangle-C_1|\psi_0|^2-C_2\langle\langle|\psi_0|^2\rangle\rangle), \tag{17}$$

where ψ_0 denotes the initial amplitude of the pump wave. When substituting in Eq. (9), one gets a nonlinear Schrödinger equation

$$i\partial_T\psi+(\chi+i\nu)\Delta_\perp\psi-\frac{k\lambda}{B_0^2}(C_1|\psi|^2+C_2\langle\langle|\psi|^2\rangle\rangle-C_1|\psi_0|^2 \\ -C_2\langle\langle|\psi_0|^2\rangle\rangle)\psi=0, \tag{18}$$

where both the diffraction coefficient

$$\chi=\frac{v_A^2}{4k\lambda}\left(1+\beta_\perp+\frac{K_1}{\rho^{(0)}v_A^2}\right) \tag{19}$$

and the damping coefficient

$$\nu=\frac{K_2}{4|k|\lambda\rho^{(0)}} \tag{20}$$

are affected by the wave-particle resonance. Here, we used the standard notation $v_A^2=B_0^2/4\pi\rho^{(0)}$ for the squared Alfvén wave velocity. Note that for an initial condition in the form of an infinitesimally perturbed plane wave, the contribution of the initial conditions in Eq. (18) reduces to a constant potential that can be eliminated by a phase shift of the solution. It is also of interest to notice that the nonlocality in the KDNLS equation induced by the wave-particle resonances does not survive at the level of the amplitude equations because of the quasi-monochromatic character of the pump.

When perturbing linearly a plane wave solution $\psi_0=\Psi_0e^{-i(k\lambda/B_0^2)(C_1+C_2)|\Psi_0|^2t}$ in the form $\psi=\psi_0(1+a+i\varphi)$, with harmonic disturbances a and φ of wavenumber K and frequency Ω, one easily gets the dispersion relation

$$\Omega=i\nu K^2\pm\sqrt{2C_1\chi k\lambda\frac{|\psi_0|^2}{B_0^2}K^2+\chi^2K^4}. \tag{21}$$

For bi-Maxwellian equilibrium distributions, ν is always negative[18] and we conclude that transverse (or filamentation) instability of a quasimonochromatic plane Alfvén wave occurs when $\chi k\lambda C_1<0$ or, assuming the plasma stable with respect to the fire-hose instability (i.e., $C_1>0$),

$$\chi_* \equiv 1+\beta_\perp + \frac{K_1}{\rho^{(0)} v_A^2} < 0. \tag{22}$$

Defining $\nu_* = K_2/\rho^{(0)} v_A^2$, we also note that the instability growth rate is governed by both the amplitude of the pump and the ratio ν_*/χ_*. The spectral range of the instability is in particular broadened when ν_*/χ_* is decreased.

IV. THE CASE OF BI-MAXWELLIAN EQUILIBRIUM DISTRIBUTIONS

It is possible to give a simple explicit formula for the quantity χ_* and provide a simple instability criterion when the equilibrium distribution functions of the different particle species are assumed bi-Maxwellian in the form,

$$F_r^{(0)} = \frac{1}{(2\pi)^{3/2}} \frac{m_r^{3/2}}{T_{\perp r}^{(0)} T_{\parallel r}^{(0)1/2}} \times \exp\left\{ -\left(\frac{m_r}{2T_{\parallel r}^{(0)}} v_\parallel^2 + \frac{m_r}{2T_{\perp r}^{(0)}} v_\perp^2 \right) \right\}. \tag{23}$$

We furthermore consider a plasma constituted of electrons (indexed by the subscript e) and of two positively charged ion species, namely, protons (indexed by p) and a species (e.g., He^{2+}) of charge Ze, indexed by α. As usual, e is the unit positive charge.

It is then useful to introduce a few additional notations that will be extensively used in the following:

$\mu_r = m_r/m_p$, mass particle ratios;

$X = n_\alpha^{(0)}/n_p^{(0)}$, ratio of α particle to proton number densities;

$a_r = T_{\perp r}^{(0)}/T_{\parallel r}^{(0)}$, temperature anisotropy factor of the particles of species r;

$\tau_r = T_{\parallel r}^{(0)}/T_{\parallel p}^{(0)}$, ratio of the parallel temperature of the particles of species r to that of the protons;

$v_{\mathrm{th},r} = \sqrt{T_{\parallel r}^{(0)}/m_r}$, thermal velocity of the particles of species r;

$v_A = B_0/\sqrt{4\pi\rho^{(0)}}$, Alfvén speed;

$\lambda^2 = v_A^2 + (p_\perp^{(0)}/\rho^{(0)}) - (p_\parallel^{(0)}/\rho^{(0)})$, squared Alfvén wave propagation velocity;

$c_r = \lambda/v_{\mathrm{th},r}$, Alfvén wave propagation velocity normalized by the thermal velocity of particles of species r;

$\beta_{\parallel p} = 8\pi p_{\parallel p}^{(0)}/B_0^2$, ratio of the parallel proton equilibrium pressure to magnetic pressure;

$\beta_\perp = 8\pi p_\perp^{(0)}/B_0^2$, ratio of the total perpendicular equilibrium pressure to magnetic pressure;

$\beta = T_e/m_p v_A^2$, squared ratio of ion acoustic velocity to Alfvén speed.

Quasineutrality provides the relation

$$n_e^{(0)} = n_p^{(0)} + Z n_\alpha^{(0)} = n_p^{(0)}(1+ZX). \tag{24}$$

It is easily checked[19] that

$$\mathcal{L}_r = -n_r^{(0)} q_r^2 \frac{1}{T_{\parallel r}^{(0)}} \mathcal{W}(c_r), \quad \mathcal{M}_r = -n_r^{(0)} q_r \frac{T_{\perp r}^{(0)}}{T_{\parallel r}^{(0)}} \mathcal{W}(c_r),$$
$$\mathcal{N}_r = -2 n_r^{(0)} \frac{T_{\perp r}^{(0)2}}{T_{\parallel r}^{(0)}} \mathcal{W}(c_r), \tag{25}$$

where

$$\mathcal{W}(c_r) = \frac{1}{\sqrt{2\pi}} \mathrm{P}\int \frac{\zeta e^{-\zeta^2/2}}{\zeta - c_r} d\zeta + \sqrt{\frac{\pi}{2}} c_r e^{-c_r^2/2} \mathcal{H}_\xi, \tag{26}$$

or[20]

$$\mathcal{W}(c_r) = 1 - c_r e^{-(c_r^2/2)} \int e^{(\zeta^2/2)} d\zeta + \sqrt{\frac{\pi}{2}} c_r e^{-c_r^2/2} \mathcal{H}_\xi. \tag{27}$$

This function is related to the plasma response function $\mathcal{R}$ used in Snyder *et al.*[21] by $\mathcal{W}(X) = \mathcal{R}(X/\sqrt{2})$. In the following, we replace all the operators by complex numbers (using the fact that $\mathcal{H}^2 = -\mathcal{I}$). For example, we denote by $\mathcal{W}_r$ the complex number obtained by replacing in Eq. (27) the Hilbert transform by the imaginary number i. One has

$$\mathcal{L} = -\frac{n_p^{(0)} e^2}{T_{\parallel p}^{(0)}} \left(\mathcal{W}_p + \frac{(1+ZX)}{\tau_e} \mathcal{W}_e + \frac{XZ^2}{\tau_\alpha} \mathcal{W}_\alpha \right), \tag{28}$$

$$\mathcal{M} = -n_p^{(0)} e (a_p \mathcal{W}_p - (1+ZX) a_e \mathcal{W}_e + XZ a_\alpha \mathcal{W}_\alpha), \tag{29}$$

$$\mathcal{N} = -2 n_p^{(0)} T_{\parallel p}^{(0)} (a_p^2 \mathcal{W}_p + (1+ZX) a_e^2 \tau_e \mathcal{W}_e + X a_\alpha^2 \tau_\alpha \mathcal{W}_\alpha). \tag{30}$$

Moreover,

$$p_\parallel^{(0)} = n_p^{(0)} T_{\parallel p}^{(0)} (1 + (1+ZX)\tau_e + X\tau_\alpha), \tag{31}$$

$$p_\perp^{(0)} = n_p^{(0)} T_{\parallel p}^{(0)} (a_p + (1+ZX) a_e \tau_e + X a_\alpha \tau_\alpha). \tag{32}$$

Noting that

$$\rho^{(0)} v_{\mathrm{th},p}^2 = n_p^{(0)} T_{\parallel p}^{(0)} (1 + (1+ZX)\mu_e + X\mu_\alpha) \tag{33}$$

and

$$\rho^{(0)} \lambda^2 = \frac{B_0^2}{8\pi} \left(2 + \frac{\beta_{\parallel p}}{n_p^{(0)} T_{\parallel p}^{(0)}} (p_\perp^{(0)} - p_\parallel^{(0)}) \right) \tag{34}$$

with $\beta_{\parallel p} = n_p^{(0)} T_{\parallel p}^{(0)} (8\pi/B_0^2)$, we can express

$$c_p^2 = \frac{2 + \beta_{\parallel p}(a_p - 1 + (1+ZX)\tau_e(a_e - 1) + X\tau_\alpha(a_\alpha - 1))}{\beta_{\parallel p}(1 + (1+ZX)\mu_e + X\mu_\alpha)}. \tag{35}$$

In addition, $c_e^2 = (\mu_e/\tau_e) c_p^2$ and $c_\alpha^2 = (\mu_\alpha/\tau_\alpha) c_p^2$. We finally obtain the parameter χ_* governing the filamentation instability in the form

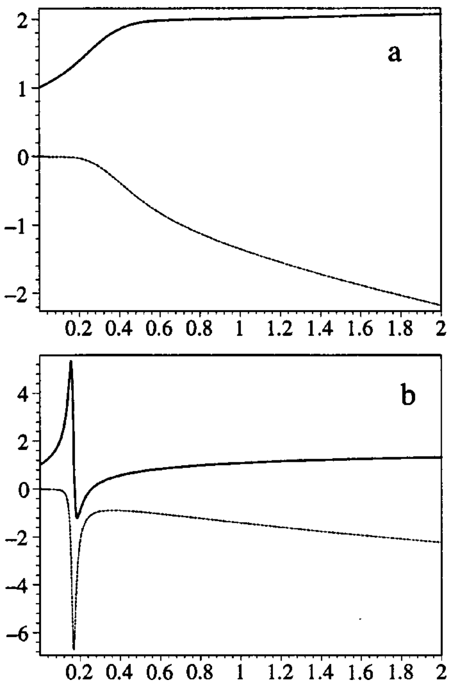

FIG. 1. Variations of χ_* (solid line) and ν_* (dashed line) as a function of $\beta_{\parallel p}$ for an equilibrium state with isotropic ion and electron temperatures ($a_p=1$, $a_e=1$), and no alpha-particles ($X=0$). In panel (a), ions and electrons have the same equilibrium temperature ($\tau_e=1$), while in panel (b), the ratio of the electron temperature to that of the protons is $\tau_e=8$. Negative values for χ_* imply filamentation instability.

$$\chi_* = 1+\beta_{\parallel p}(a_p+(1+ZX)a_e\tau_e+Xa_\alpha\tau_\alpha) + \frac{\beta_{\parallel p}}{2n_p^{(0)}T_{\parallel p}^{(0)}}\Re(\mathcal{N}-\mathcal{M}^2\mathcal{L}^{-1}), \tag{36}$$

with $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ defined in Eqs. (28)–(30). As previously mentioned, instability occurs for $\chi_*<0$.

V. FILAMENTATION CRITERION IN A FEW TYPICAL REGIMES

The parameter space being very large, the behavior of the function χ_* cannot be simply summarized. We shall here focus on a few examples that illustrate the variations of the instability range with the medium parameters, in particular with the electron to proton temperature ratio and the electron temperature anisotropy.

Figure 1(a) displays the coefficients χ_* and ν_* as functions of $\beta_{\parallel p}$ for a proton–electron plasma with equal temperatures ($a_p=a_e=1$, $\tau_e=1$, $X=0$). In this case, χ_* is always positive and no filamentation takes place. In Fig. 1(b), a similar plot is shown with $\tau_e=8$, showing that increasing the electron temperature with respect to that of the protons leads to the apparition of a small range of values of $\beta_{\parallel p}$ for which $\chi_*<0$ and filamentation occurs. This observation suggests to consider the limit $\tau_e\gg 1$ of a plasma with hot electrons and cold protons. We further assume that the Alfvén speed $v_A=B_0/\sqrt{4\pi\rho^{(0)}}$ satisfies $v^2_{\mathrm{th},p}\ll v_A^2\ll v^2_{\mathrm{th},e}$, which implies $(m_e/m_p)\ll\beta\ll(T_e/T_p)$ and corresponds to adiabatic protons and isothermal electrons. Here, β refers to the squared ratio of the ion acoustic to the Alfvén speed which also rewrites $\beta=\tau_e/c_p^2$. Alpha particles are assumed to be absent ($X=0$). Noting that in this asymptotics, $\mathcal{W}_p\approx -1/c_p^2$ and $\mathcal{W}_e\approx 1+i\sqrt{(\pi/2)}c_e$, we find

$$\frac{1}{2n_p^{(0)}T_{\parallel p}^{(0)}}\Re(\mathcal{N}-\mathcal{M}^2\mathcal{L}^{-1}) \approx -\tau_e a_e^2\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)\times\left(1+\frac{\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)}{2\left(\beta-\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)\right)}\right). \tag{37}$$

Even though $c_e\ll 1$, the electron damping terms have been retained to avoid the singularity at $\beta=1$. Note that (using $\mu_e\ll 1$)

$$\beta_{\parallel p}\approx\frac{2\beta}{\tau_e[1+\beta(1-a_e)]}. \tag{38}$$

It follows that when $a_e>1$, $\beta_{\parallel p}$ can vary from 0 to infinity while β is limited to the range $[0,(a_e-1)^{-1}]$, which makes the use of $\beta_{\parallel p}$ as the independent variable preferable in the general case. In the present regime, the parameter χ_* becomes

$$\chi_*=1+\frac{2\beta a_e}{1+\beta(1-a_e)}\left(1-a_e\Re\left(\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)\times\left(1+\frac{\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)}{2\left(\beta-\left(1+i\sqrt{\frac{\pi}{2}}c_e\right)\right)}\right)\right)\right), \tag{39}$$

which reduces to

$$\chi_*=\frac{1}{1-\beta} \tag{40}$$

when the electron temperature is isotropic ($a_e=1$) and the electron Landau damping neglected ($c_e=0$). The usual fluid limit is thus recovered provided β is correctly interpreted.[22] In this limit, filamentation instability takes place for $\beta>1$ or equivalently for $\beta_{\parallel p}>2/\tau_e$.

Let us now consider the influence of the electron temperature anisotropy a_e for a finite and fixed value of the electron to ion temperature ratio $\tau_e=8$. Comparing Figs. 1(b) and 2(a), we note that when a_e is taken moderately larger than unity [in Fig. 2(b), $a_e=1.8$], filamentation instability exists in a wider range of values of β, with a larger growth rate. In this case, the ratio ν_*/χ_* tends to 0.45 as $\beta_{\parallel p}$ tends to infinity (it is to be noted that the limit value of this ratio does not vary monotonically with τ_e but is, for example, equal to 0.12 when $\tau_e=20$). When a_e is further increased, filamentation can disappear [as in Fig. 2(b), where $a_e=2.5$] and reappear [as in Fig. 2(c), where $a_e=4.5$]. In the latter

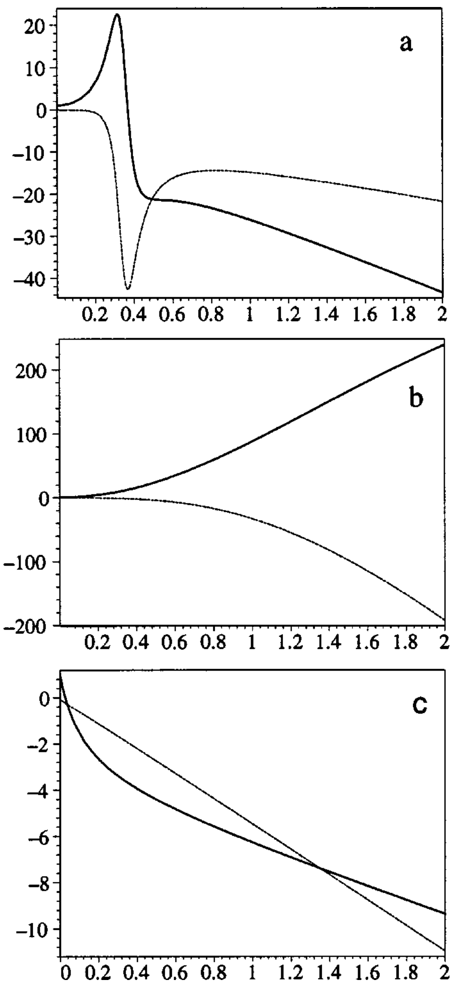

FIG. 2. Same as Fig. 1 when $a_p=1$, $\tau_e=8$, and $X=0$, for different anisotropy degree of the equilibrium electron temperature [$a_e=1.8$ in panel (a), $a_e=2.5$ in panel (b), and $a_e=4.5$ in panel (c)]. In the latter case, filamentation instabilbity occurs for $\beta>0.028$.

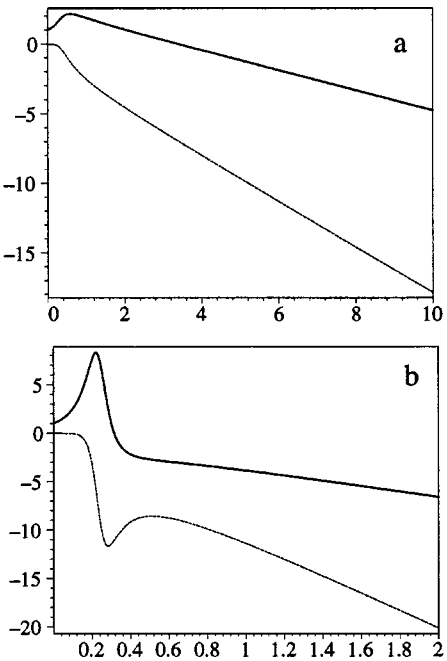

FIG. 3. Same as Fig. 1 with, in panel (a), $a_p=1$, $\tau_e=1$, $X=0$, and $a_e=1.8$ (corresponding to isotropic proton and anisotropic electron equilibrium temperatures), and in panel (b) $a_p=3.25$, $a_e=1.27$, $a_\alpha=3.25$, $\tau_e=5.5$, $\tau_\alpha=4$, $X=0.1$, $\mu_\alpha=4$, and $Z=2$ (corresponding to anisotropic temperatures for all species, with hotter electrons and alpha-particles whose number density is 10% that of the protons).

case, filamentation instability exists for $\beta_{\parallel p}>0.028$, to be compared with the value 1 obtained in the fluid limit.

The complicated behavior observed above for $\tau_e=8$, simplifies when τ_e is of order unity. In this case, a qualitative argument can be presented. In the absence of α particles ($X=0$) and for a parameter $\beta_{\parallel p}$ of order unity or larger, one easily sees that $\mathcal{W}_e \approx 1$, and that both the real and imaginary parts of $\mathcal{W}_p$ are of order unity. The dominant contribution to the real part K_1 of the complex number associated with the kinetic operator $\mathcal{K}$, comes from $\mathcal{N}$ and one thus concludes, using (30) and (36), that χ_* becomes negative when $\beta_{\parallel p}$ exceeds a critical value β_c, provided $a_e>1$. The existence of such a critical value for β is illustrated in Fig. 3(a) that displays χ_* and ν_* as a function of $\beta_{\parallel p}$ for $a_e=1.8$, $\tau_e=1$ and $X=0$. Similar graphs are shown in Fig. 3(b) for parameters typical of the solar wind,[17] $a_p=3.25$, $a_e=1.27$, $a_\alpha=3.25$, $\tau_e=5.5$, $\tau_\alpha=4$, $X=0.1$, $\mu_\alpha=4$, and $Z=2$. In this regime, filamentation takes place for $\beta_{\parallel p}>0.315$.

The value of β_c decreases when τ_e increases, a_e being fixed at a value of order unity, or with a_e, when τ_e is of order one. This variation of β_c as function of a_e and τ_e are respectively illustrated in Fig. 4(a) (with $\tau_e=1$) and in Fig. 4(b) (with $a_e=1.5$) both assuming $a_p=1$. Finally, we show in Fig. 5(a) the variation of β_c as a function of a_p (for $a_e=1.5$, $\tau_e=3$, and $X=0$) and in Fig. 5(b) its variation as a function of X (with $a_e=1.5$, $a_p=1$, and $\tau_e=3$). Proton an-

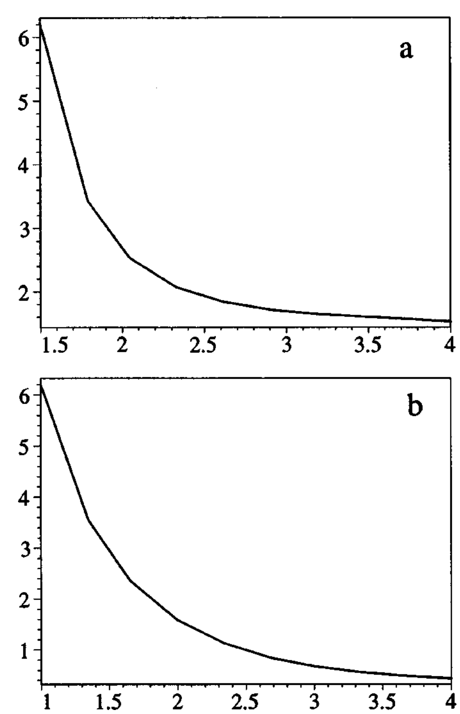

FIG. 4. Variation of the critical value β_c for the onset of filamentation instability vs the equilibrium electron temperature anisotropy a_e for $a_p=1$, $\tau_e=1$, $X=0$ (panel a) and vs the ratio of the electron to proton equilibrium parallel temperature τ_e for $a_p=1$, $a_e=1.5$, $X=0$ (panel b).

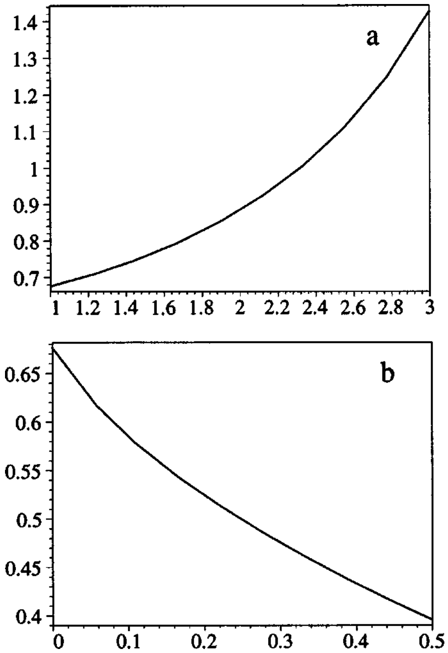

FIG. 5. Variation of β_c as a function of the anisotropy degree of the proton equilibrium temperature a_p for $a_e=1.5$, $\tau_e=3$, $X=0$ (panel a) and as a function of the relative alpha-particle number density, measured by X for $a_p=1$, $\tau_e=3$, $a_e=1.5$ (panel b).

isotropy tends to inhibit filamentation, while the presence of alpha-particles broadens the instability criterion, at least for the considered case. An opposite trend has been observed in the situation of Fig. 2(c).

VI. CONCLUSION

In this paper we have shown that a circularly polarized quasimonochromatic Alfvén wave propagating along an ambient magnetic field in a collisionless plasma can be subject to a filamentation instability, provided temperature ratios are in appropriate parameter ranges. The analysis is performed on a KDNLS equation systematically derived from the Vlasov–Maxwell equations under the assumptions that the Alfvén wave amplitude is sufficiently small and its wavelength long compared to the ion inertial length. For purely transverse large-scale modulation, the wave amplitude is shown to obey a two-dimensional nonlinear Schrödinger equation with an extra linear dissipative term originating from the Landau damping. Absolute filamentation instability, which is investigated under the assumption of bi-Maxwellian equilibrium distribution functions, usually exists for β larger than a critical value that, in some instances, can be significantly smaller than one. The range of existence of the instability is in general enlarged as the ratio of electron to ion temperatures and/or the electron temperature anisotropy are increased. This behavior can be physically understood by noting that filamentation involves the coupling of the Alfvén waves with perpendicular gradients of perpendicular pressure fluctuations with finite parallel wavenumber [term proportional to $\partial_\perp \widetilde{P}$ in Eq. (1)]. It is thus not surprising that the instability is strengthened when the electron to proton temperature ratio is large (a case where ion acoustic waves are only weakly damped), and when perpendicular (electron) pressure is enlarged.

Several questions deserve further studies. It is in particular important to investigate the role of deviations from bi-Maxwellian equilibrium distributions, as in many space plasmas such deviations are quite significant and were shown for example to have a significant effect on the mirror instability threshold.[23] Transverse instabilities seem to be of interest in the context of auroral plasmas for which β is very small. In that situation, filamentation is expected to develop in a convective regime.[9] The analysis performed in this paper is not appropriate for this situation and one should rather start from the Hall Landau-fluid model derived elsewhere.[24,25] The nonlinear development of the instability is also interesting to investigate. In situations we have considered, the ratio ν_*/χ_* is never really small so that Landau damping is expected to arrest wave collapse quite early, making the envelope description valid throughout the evolution. A quantitative estimate of the saturation level in terms of the strength of the Landau damping requires a parametric study of Eq. (18), that is currently underway. Finally the question arises of the persistence of the filamentation instability in the regime of Alfvén wave turbulence. Direct simulations of the Hall Landau-fluid model[24] can provide a convenient framework to address this issue.

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