

Figures of lectures 4 and 5

Geodesics and images in the Kerr metric

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<https://relativite.obspm.fr/blackholes/paris25/>

PSL graduate program in Physics / Doctoral School PIF
ENS, Paris, France
21 and 28 May 2025

Home page for the lectures

<https://relativite.obspm.fr/blackholes/paris25/>

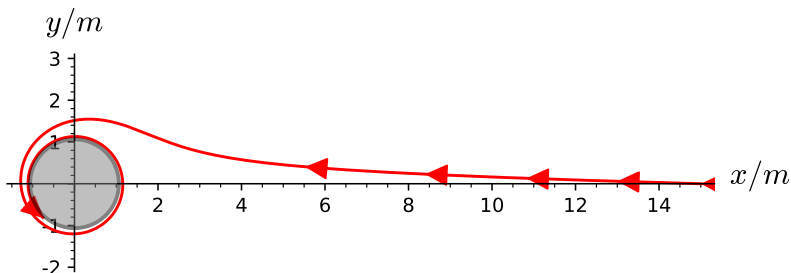
includes

- these slides
- the lecture notes (draft)
- some SageMath notebooks

Prerequisite

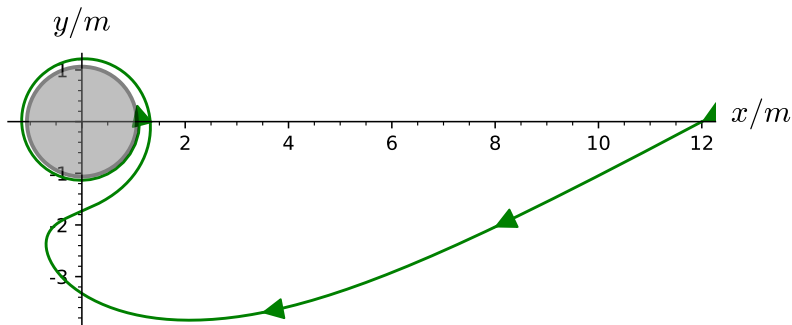
An introductory course on general relativity

Lense-Thirring effect



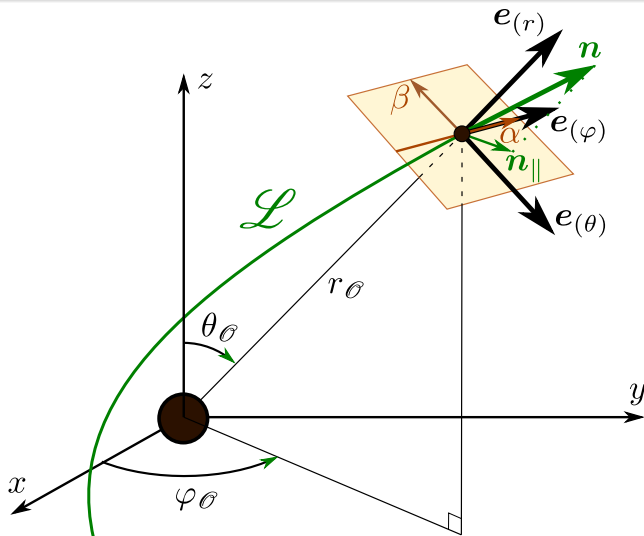
Trajectory in the equatorial plane of an incoming timelike geodesic with $E = \mu$, $L = 0$ and $Q = 0$, plunging into a Kerr black hole with $a = 0.998 m$.

Lense-Thirring effect



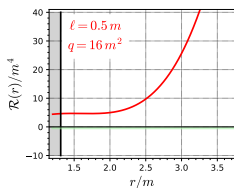
Trajectory in the equatorial plane of an incoming null geodesic with $L = -6E < 0$ and $Q = 0$, plunging into a Kerr black hole with $a = 0.998 m$.

Screen coordinates (α, β) of remote observer

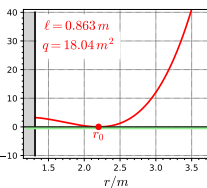


Impact of a null geodesic $\mathcal{L}$ onto the screen of a remote observer $\mathcal{O}$

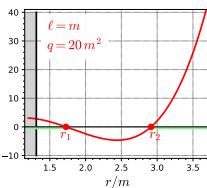
Quartic polynomial $\mathcal{R}(r)$ in $\mathcal{M}_I$



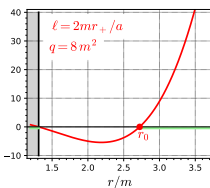
(a)



(b)



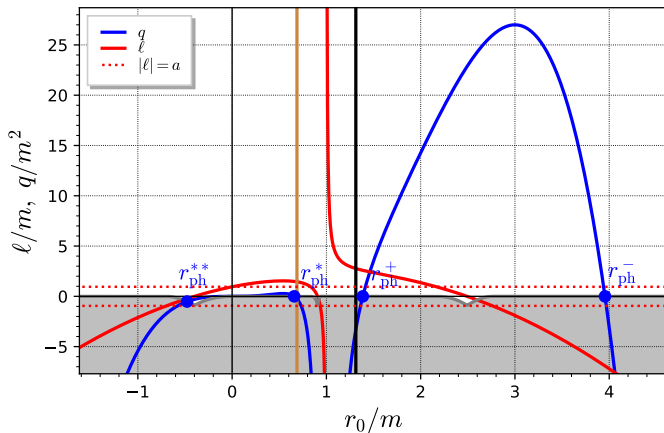
(c)



(d)

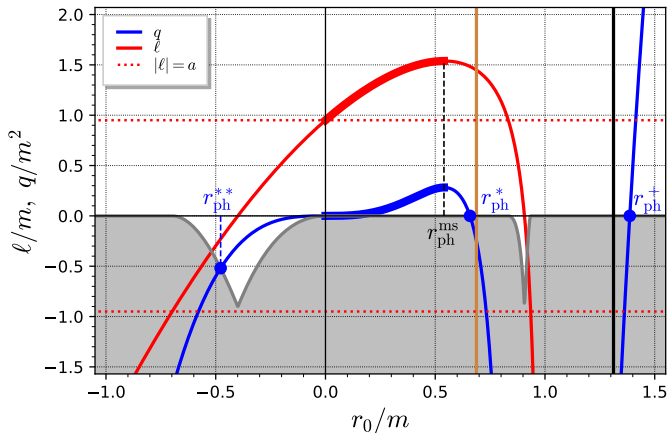
Quartic polynomial $\mathcal{R}(r)$ in the region $\mathcal{M}_I$ for four values of (ℓ, q) and for $a = 0.95m$

Existence of spherical photon orbits



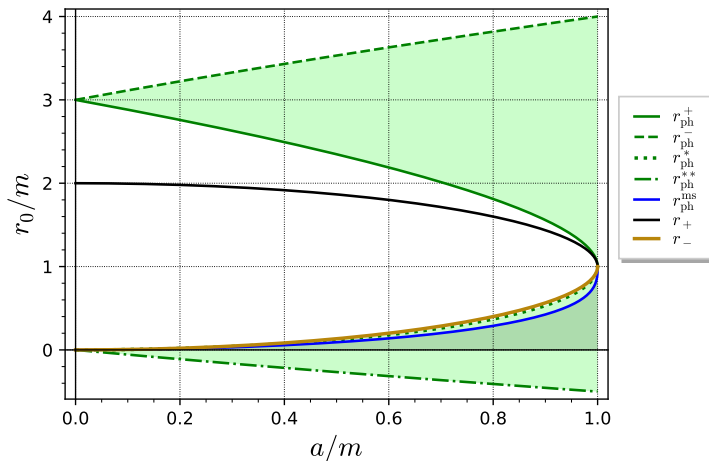
Functions $\ell_c(r_0)$ (in red) and $q_c(r_0)$ (in blue) giving the reduced angular momentum and reduced Carter constant of a spherical photon orbit of radius r_0 for $a = 0.95m$. Values of q in the grey zone are unphysical.

Existence of spherical photon orbits



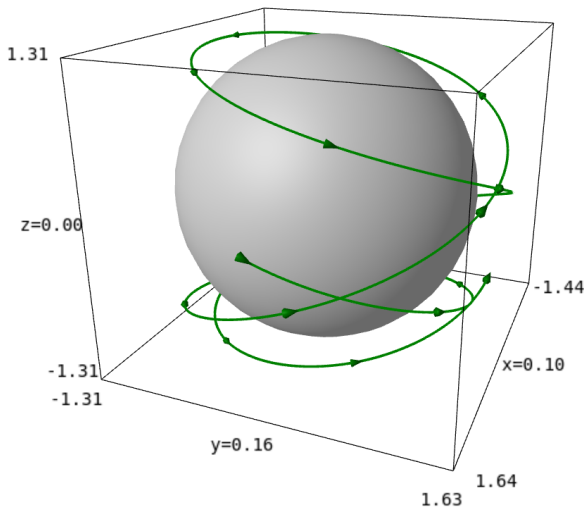
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Range of spherical photon orbits



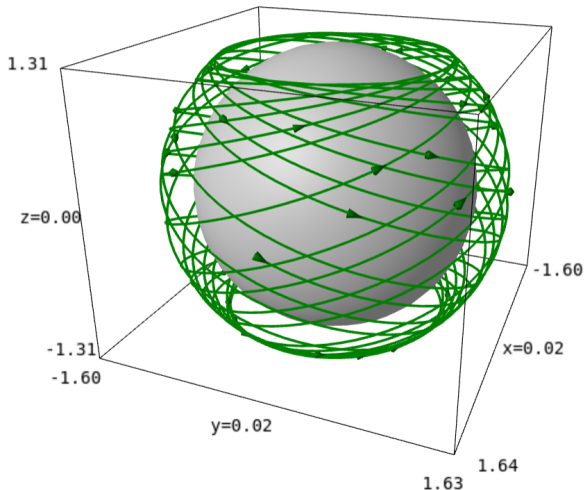
Domain of existence of spherical photon orbits in the (a, r_0) plane (in green). The shaded area correspond to stable spherical orbits.

Spherical photon orbit at $r_0 = 1.6 m$



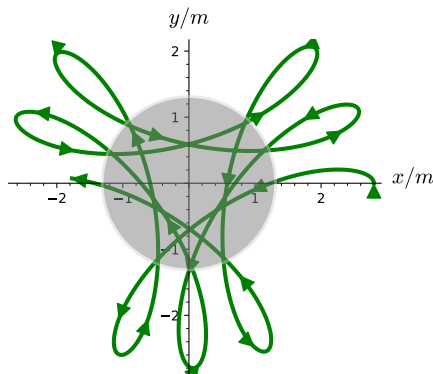
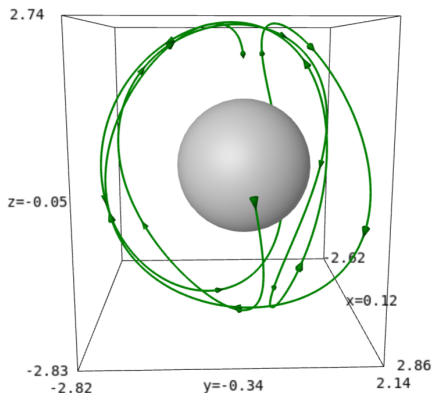
Spherical photon orbit at $r_0 = 1.6 m$ around a Kerr black hole with $a = 0.95 m$. Plot for $0 < \lambda < 7.7 m/E$.

Spherical photon orbit at $r_0 = 1.6 m$



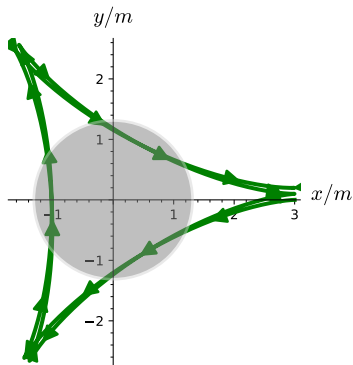
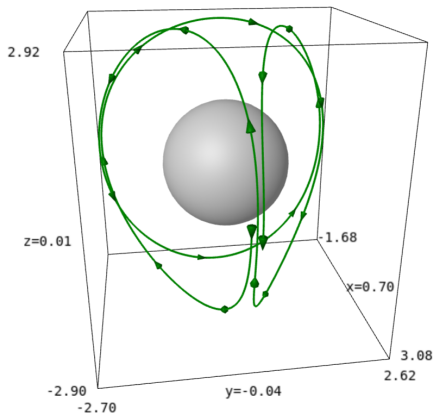
Spherical photon orbit at $r_0 = 1.6 m$ around a Kerr black hole with $a = 0.95 m$. Plot for $0 \leq \lambda \leq 70 m/E$.

Spherical photon orbit at $r_0 = 2.8 m$



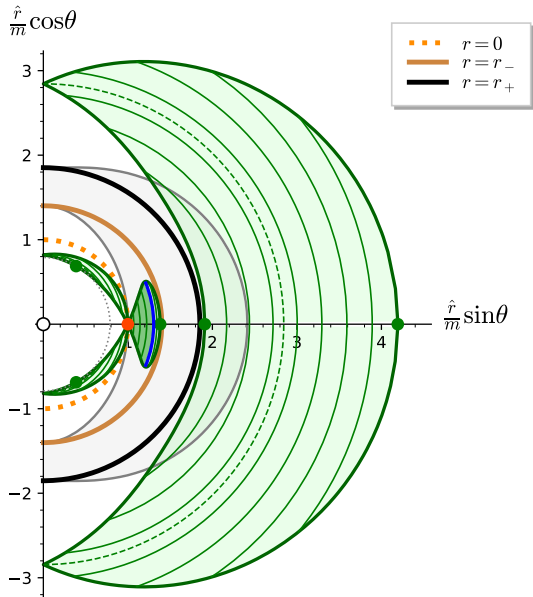
Spherical photon orbit at $r_0 = 2.8 m$ around a Kerr black hole with $a = 0.95 m$

Spherical photon orbit at $r_0 = 3m$



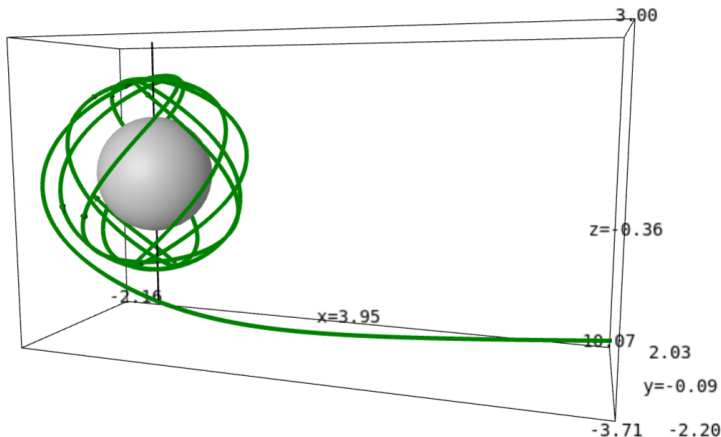
Spherical photon orbit at $r_0 = 3m$ around a Kerr black hole with $a = 0.95m$

Photon region



Trace of the photon region
(pale and dark green areas) in
a meridional plane
 $(t, \varphi) = \text{const}$ of a Kerr
spacetime with $a = 0.95 m$

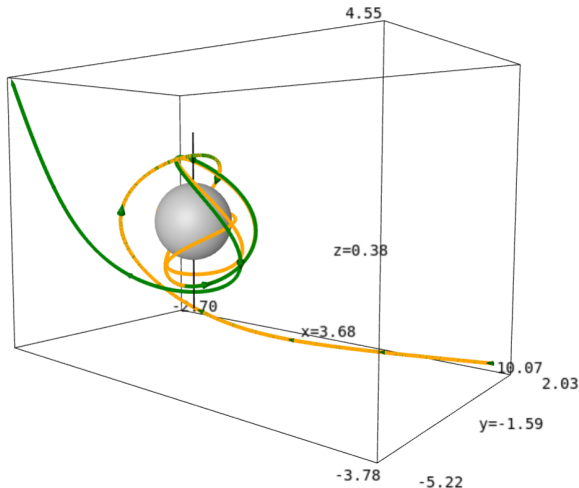
Critical null geodesic



Critical null geodesic of parameter $r_0 = 2.2 m$ in a Kerr spacetime with $a = 0.95 m$. For an interactive 3D view, see the SageMath notebook

https://nbviewer.org/github/egourgoulhon/BHlectures/blob/master/sage/Kerr_null_geod_plots.ipynb

Near-critical null geodesics

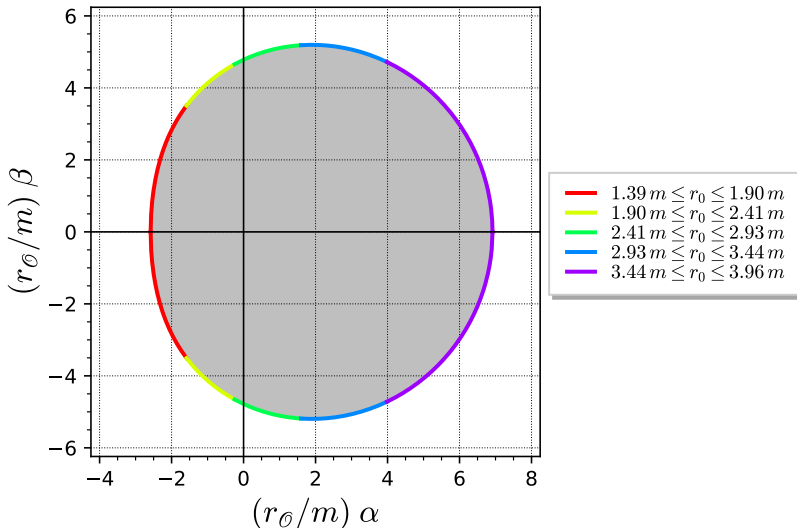


Both geodesics have $q = q_c(r_0)$ with $r_0 = 2.2 m$ ($a = 0.95 m$)
green: $\ell = 1.0001 \ell_c(r_0)$, orange: $\ell = 0.9999 \ell_c(r_0)$

<https://nbviewer.org/github/egourgoulhon/BHlectures/blob/master/sage/>

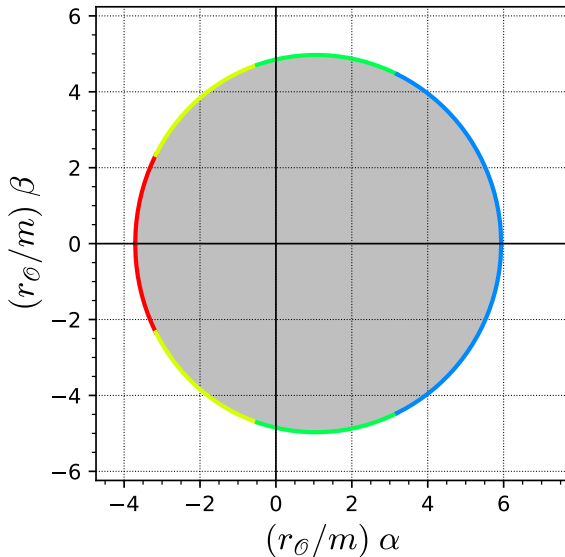
Critical curve on the observer's screen

$$a = 0.95 m, \theta_{\mathcal{O}} = \pi/2$$



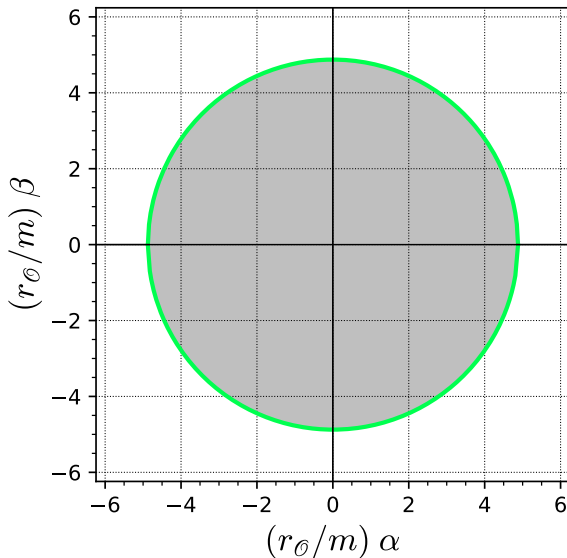
Critical curve on the observer's screen

$$a = 0.95 m, \theta_{\mathcal{O}} = \pi/6$$



Critical curve on the observer's screen

$$a = 0.95 m, \theta_{\mathcal{O}} = 0$$

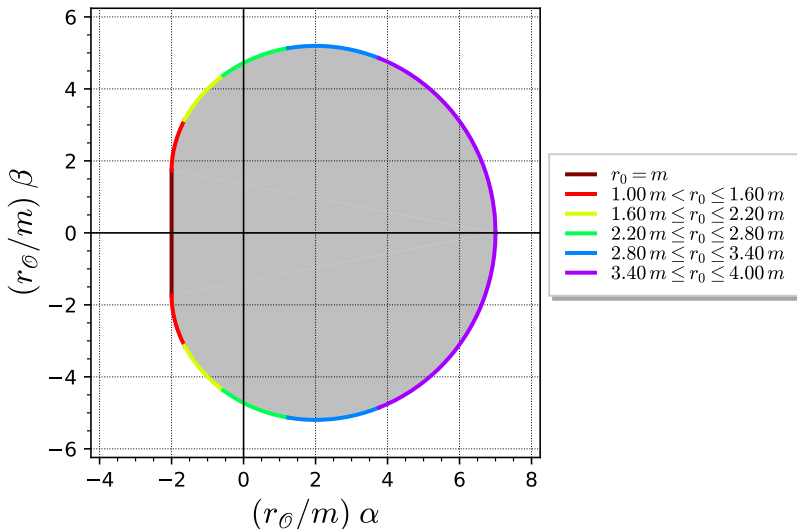


Scale factor m/r_θ

	Sgr A*	M87*	M31*	Cyg X-1
$m [M_\odot]$	$4.1 \cdot 10^6$	$6.2 \cdot 10^9$	$1.5 \cdot 10^8$	15
r_θ [kpc]	8.12	$1.67 \cdot 10^4$	$7.6 \cdot 10^2$	1.86
m/r_θ	$2.4 \cdot 10^{-11}$	$1.8 \cdot 10^{-11}$	$9.4 \cdot 10^{-12}$	$3.9 \cdot 10^{-16}$
$m/r_\theta [\mu\text{as}]$	5.0	3.7	1.9	$8.0 \cdot 10^{-5}$

Critical curve for an extremal Kerr black hole

$$a = m, \theta_{\mathcal{O}} = \pi/2$$



Critical curve and spin parameter a

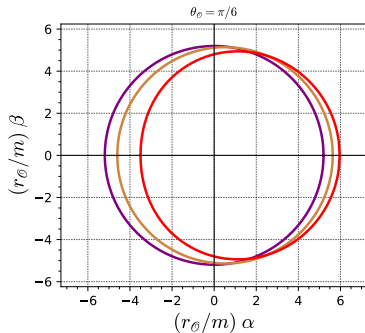
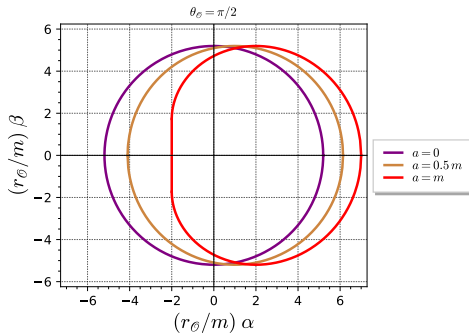
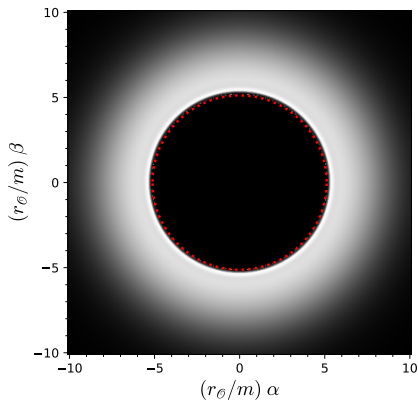


Image of an accretion disk

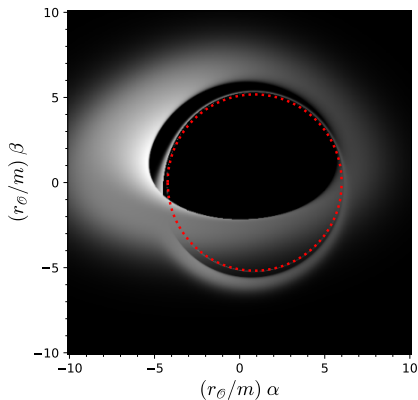
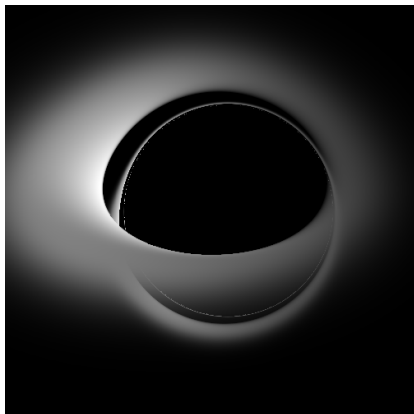
$$a = 0.5 m, \theta_o = 0$$



Computation with Gyoto ray tracing code (<https://gyoto.obspm.fr/>)

Image of an accretion disk

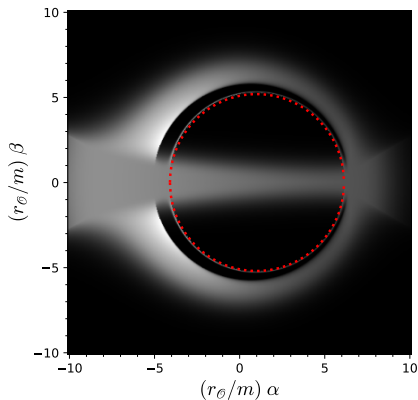
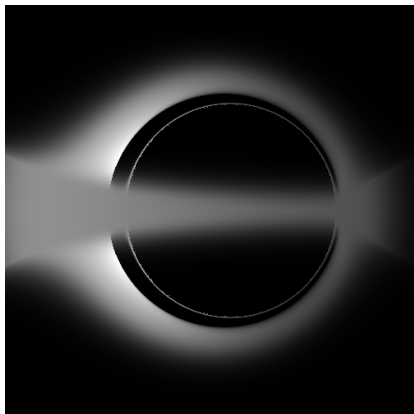
$$a = 0.5m, \theta_{\mathcal{O}} = \pi/3$$



Computation with Gyoto ray tracing code (<https://gyoto.obspm.fr/>)

Image of an accretion disk

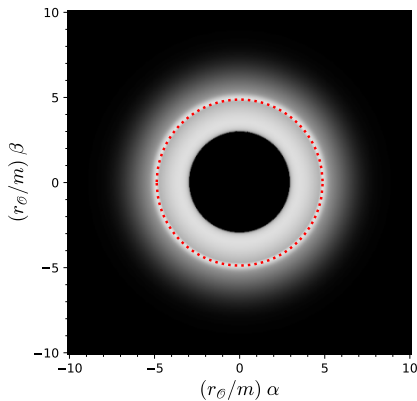
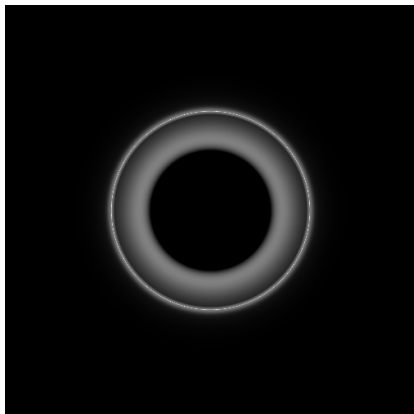
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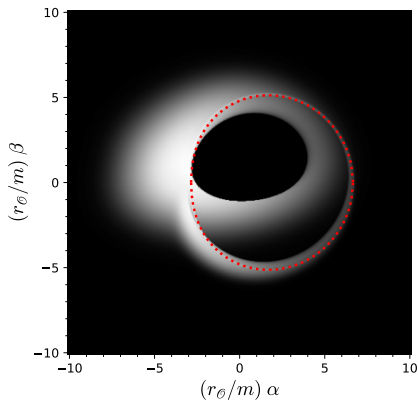
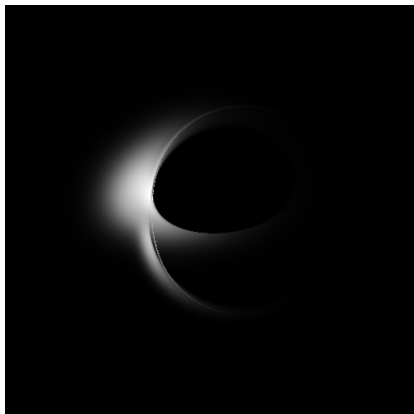
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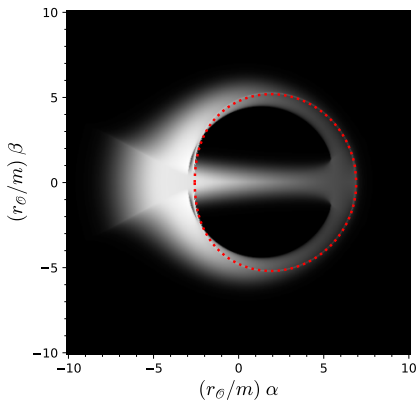
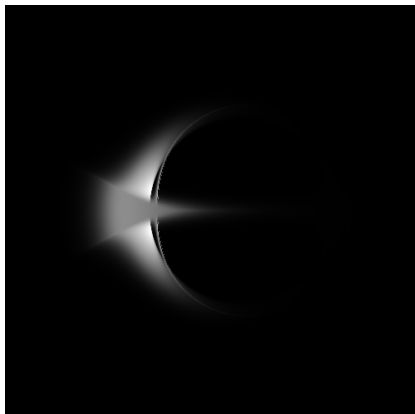
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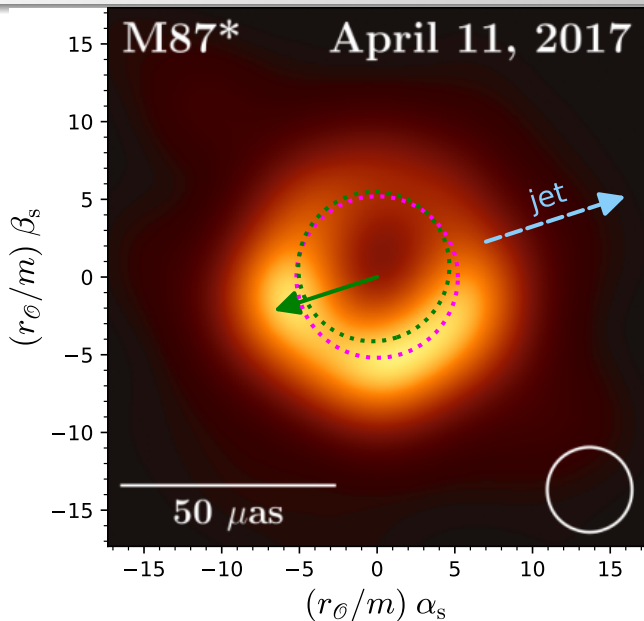
Image of an accretion disk

$$a = 0.95 m, \theta_{\mathcal{O}} = \pi/2$$



Computation with Gyoto ray tracing code (<https://gyoto.obspm.fr/>)

EHT image of M87*



$$\theta_{\mathcal{O}} = 163^\circ$$

- magenta: $a = 0$
- green: $a = m$