Figures of lecture 7

Quasi-local approaches and Penrose's singularity theorem

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https://relativite.obspm.fr/blackholes/paris23/

PSL graduate programs in Physics and in Astrophysics
ENS, Paris, France
20 June 2023

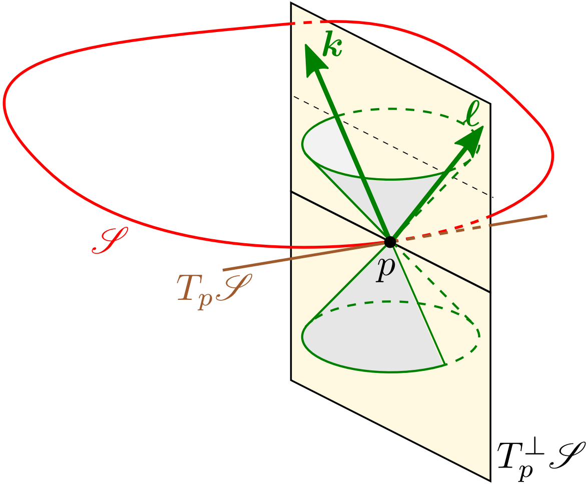
Home page for the lectures

https://relativite.obspm.fr/blackholes/paris23

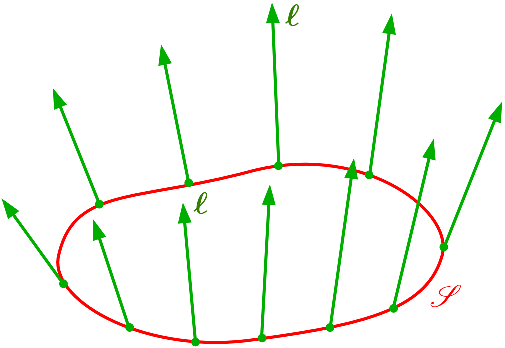
includes

- the lecture notes (draft)
- some SageMath notebooks
- these slides

Orthogonal complement of the tangent space to a spacelike $(n-2)$-surface



Expansion along a null normal



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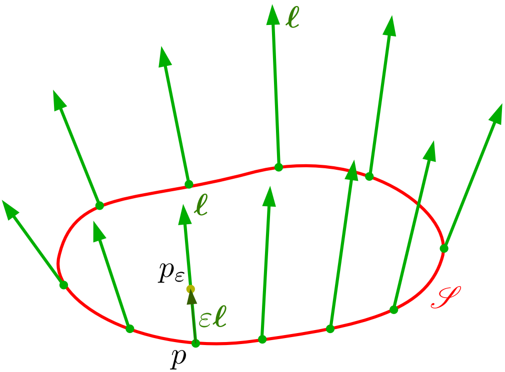

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2. ε being a small parameter, displace the point p by the vector $\varepsilon\boldsymbol{\ell}$ to the point p_ε

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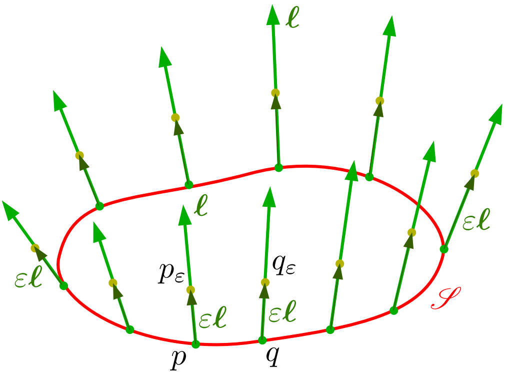

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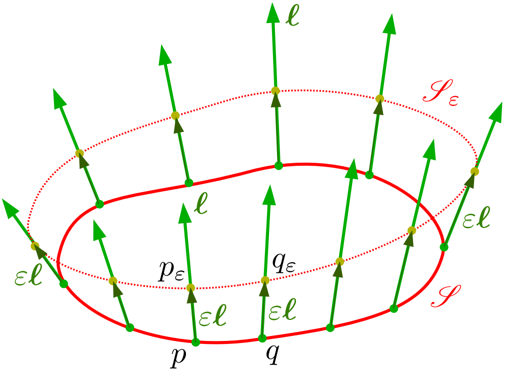

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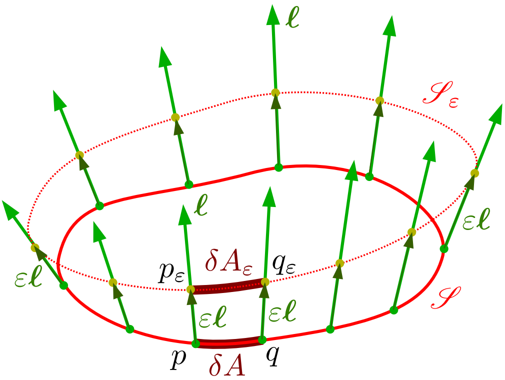

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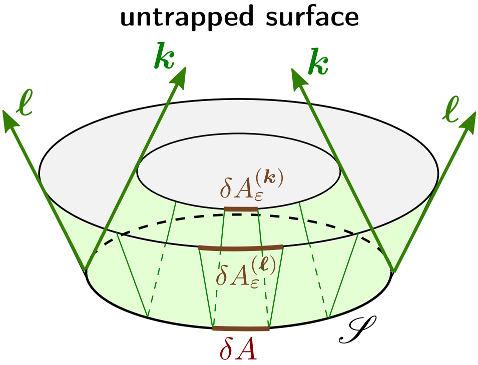
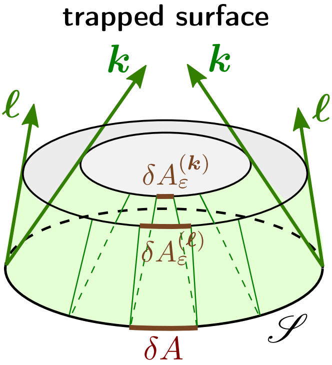
At each point, the **expansion along** $\boldsymbol{\ell}$ is defined from the relative change of the area element δA:

$$\theta_{(\boldsymbol{\ell})} := \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\frac{\delta A_\varepsilon - \delta A}{\delta A} = \mathcal{L}_{\boldsymbol{\ell}} \ln\sqrt{q} = q^{\mu\nu}\nabla_\mu \ell_\nu$$

Trapped surfaces



$\theta_{(\boldsymbol{k})} < 0$ and $\theta_{(\boldsymbol{\ell})} > 0$



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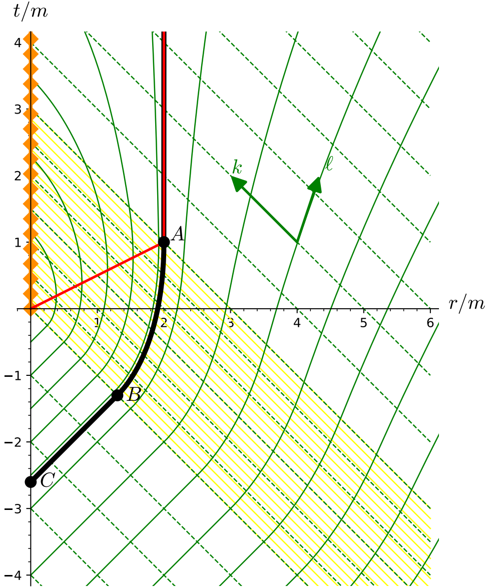
Trapped surfaces in the Vaidya collapse

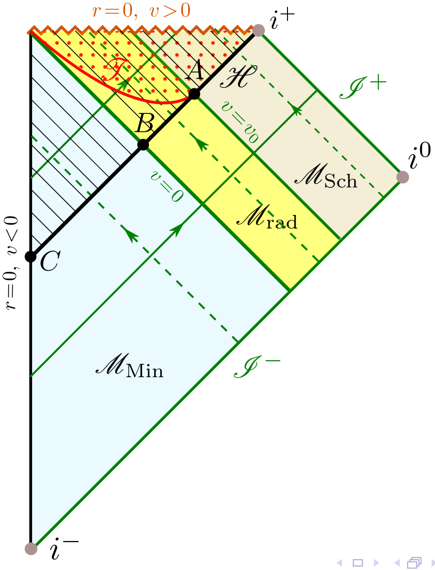


Collapse of shell of electromagnetic radiation

r = areal radius

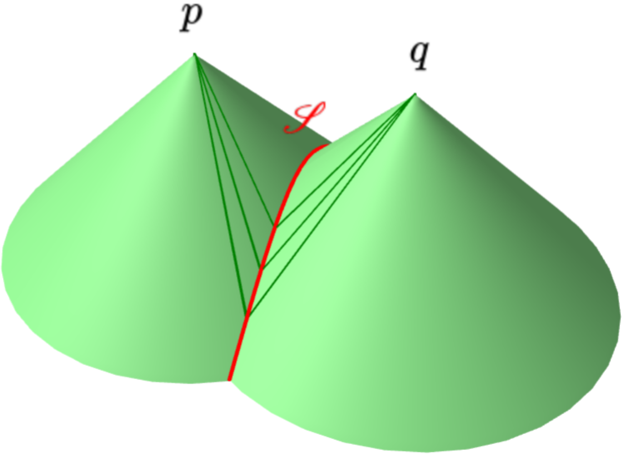
SageMath notebook: https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Vaidya.ipynb

Carter-Penrose diagram of the Vaidya collapse



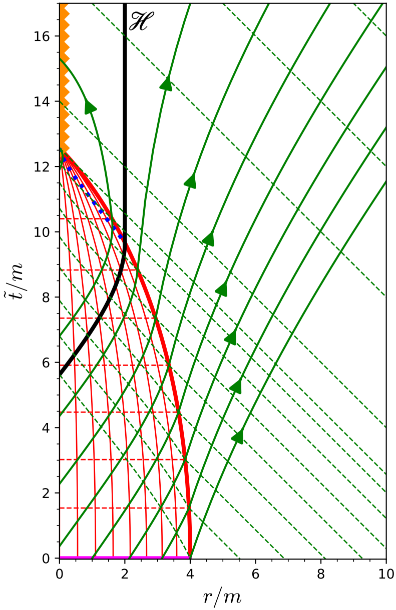
Importance of the compactness hypothesis

Intersection of two past null cones in Minkowski spacetime



https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/loc_cone_intersect.ipynb

Trapped surfaces in the Oppenheimer-Snyder collapse



Collapse of a ball of pressureless matter (dust) initially at rest

r = areal radius

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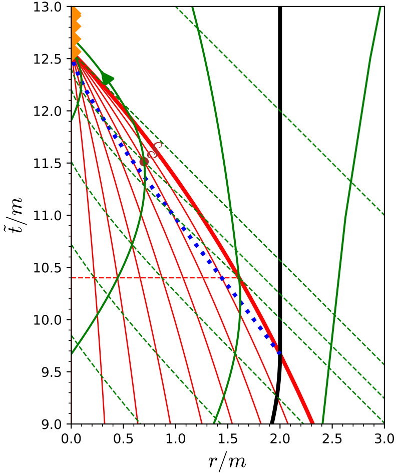

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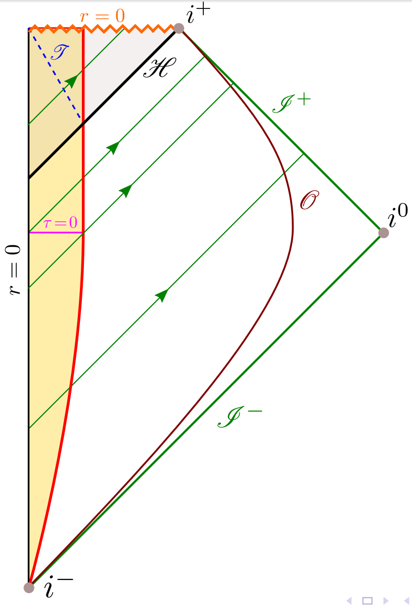
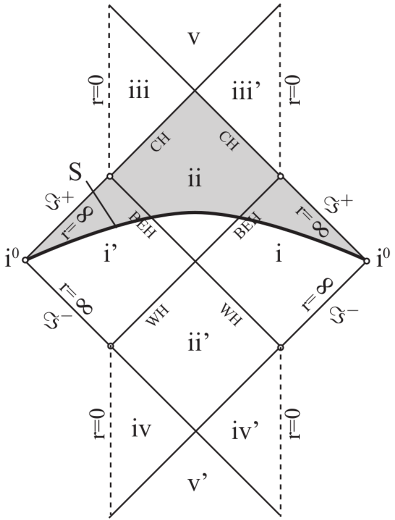


Carter-Penrose diagram of Bardeen's regular black hole



[Maeda, JHEP **2022**, 108 (2022)]