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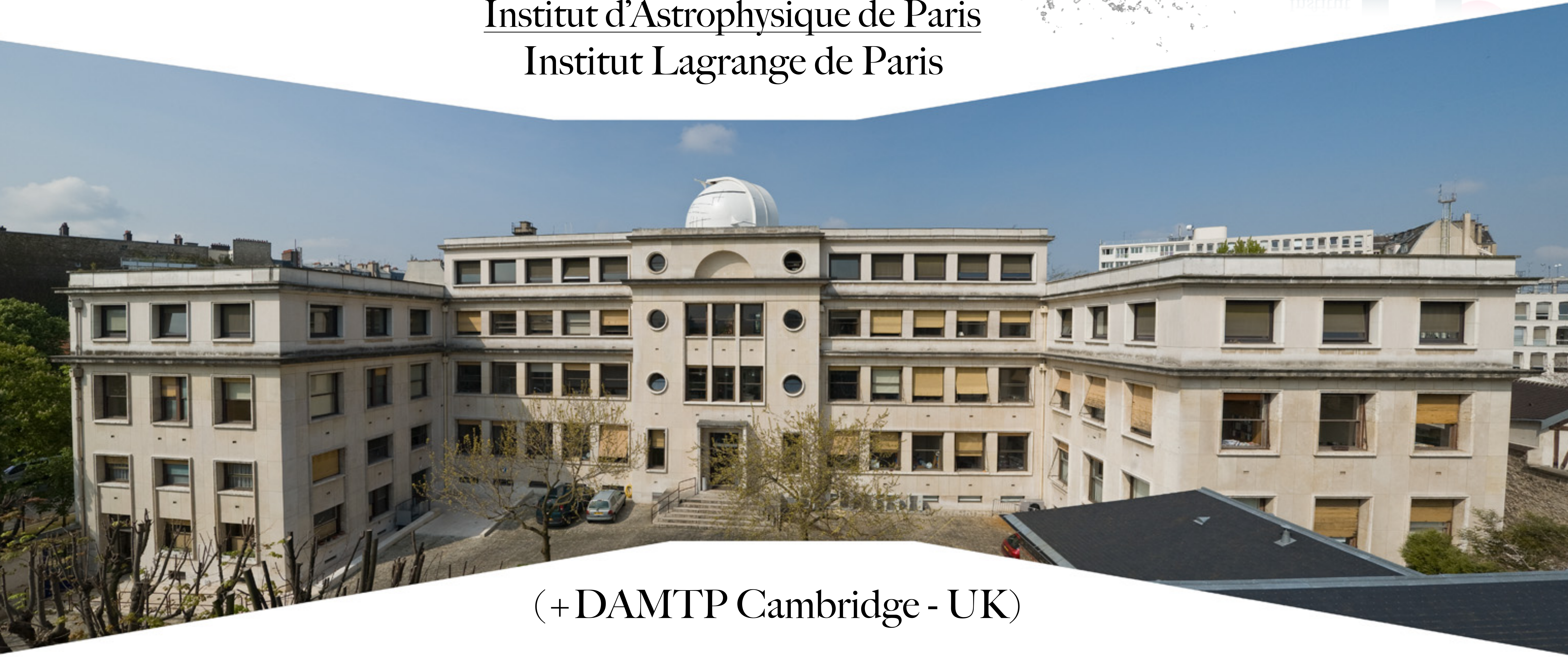


(+DAMTP Cambridge - UK)

Quantum avoidance of the Friedmann singularity

LUTh - 22/11/2018

Motivations: (quantum) cosmology

Homogeneous & isotropic metric (FLRW) $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathscr{K}r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$

Hubble rate $H \equiv \frac{\dot{a}}{a}$

Matter component: perfect fluid $T_{\mu\nu} = p g_{\mu\nu} + \left(\rho + p\right) u_\mu u_\nu$

Equation of state $p = w\rho$ → $\begin{cases} w = 0 & \textit{dust} \\ w = \frac{1}{3} & \textit{radiation} \end{cases}$

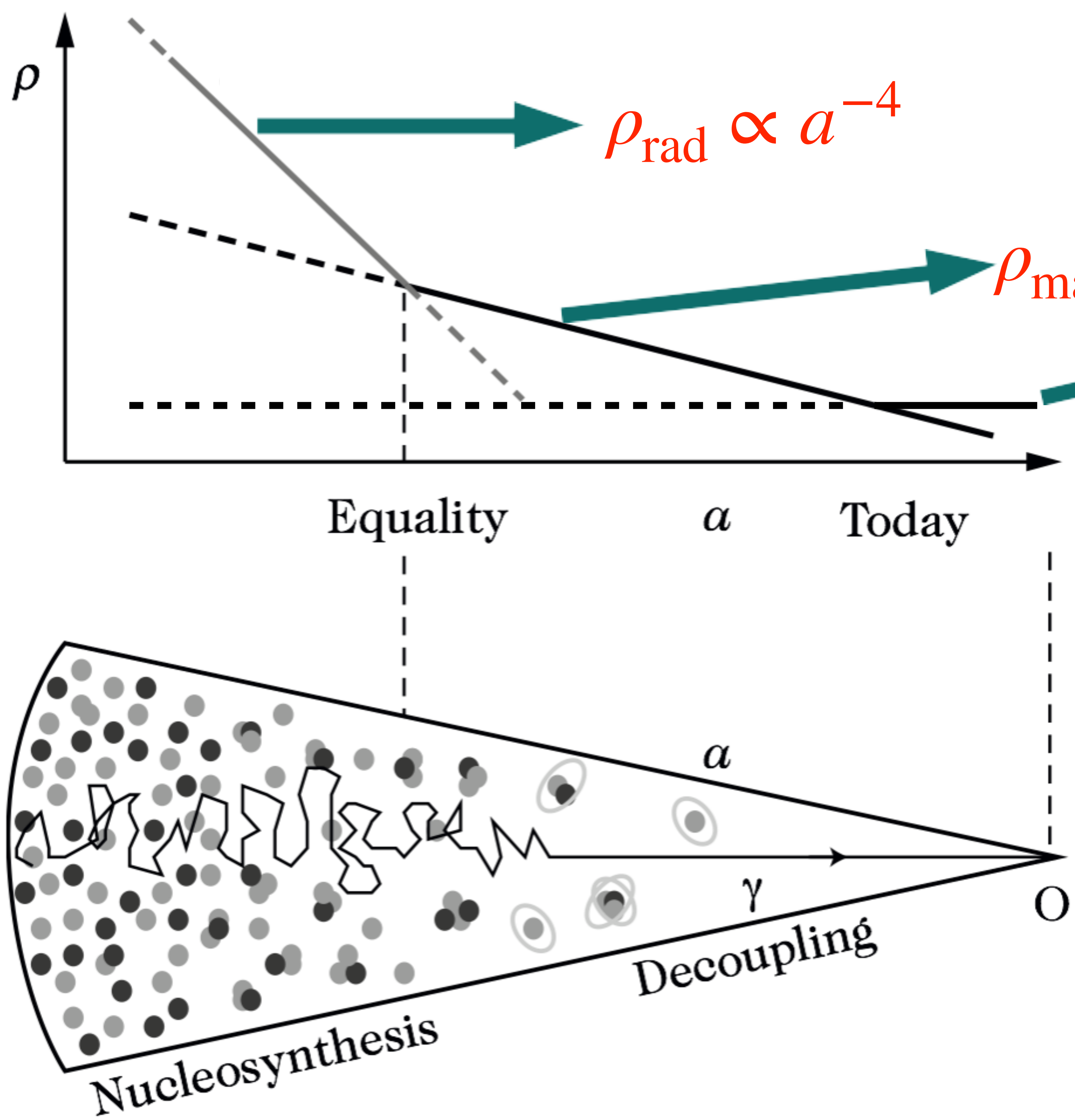
+cosmological constant = Einstein/Friedmann equations

$$\begin{cases} H^2 + \frac{\mathscr{K}}{a^2} = \frac{1}{3}\left(8\pi G_{\mathrm{N}}\rho + \Lambda\right) \\ \frac{\ddot{a}}{a} = \frac{1}{3}\left[\Lambda - 4\pi G_{\mathrm{N}}\left(\rho + 3p\right)\right] \end{cases}$$

Particular solution: dust and radiation

integrate conservation equation

$$\rho\left[a\left(t\right)\right] = \rho_{\text{ini}} \exp\left\{-3\int\left[1+w(a)\right]\mathrm{d}\ln a\right\} \underset{w\to\text{cst}}{=} \rho_{\text{ini}}\left(\frac{a}{a_{\text{ini}}}\right)^{-3(1+w)}$$



$\rho_{\text{rad}} \propto a^{-4}$

$\rho_{\text{mat}} \propto a^{-3}$

$\rho_{\Lambda} \propto a^{0}$

Phenomenologically valid description for 14 Gyrs!!!!

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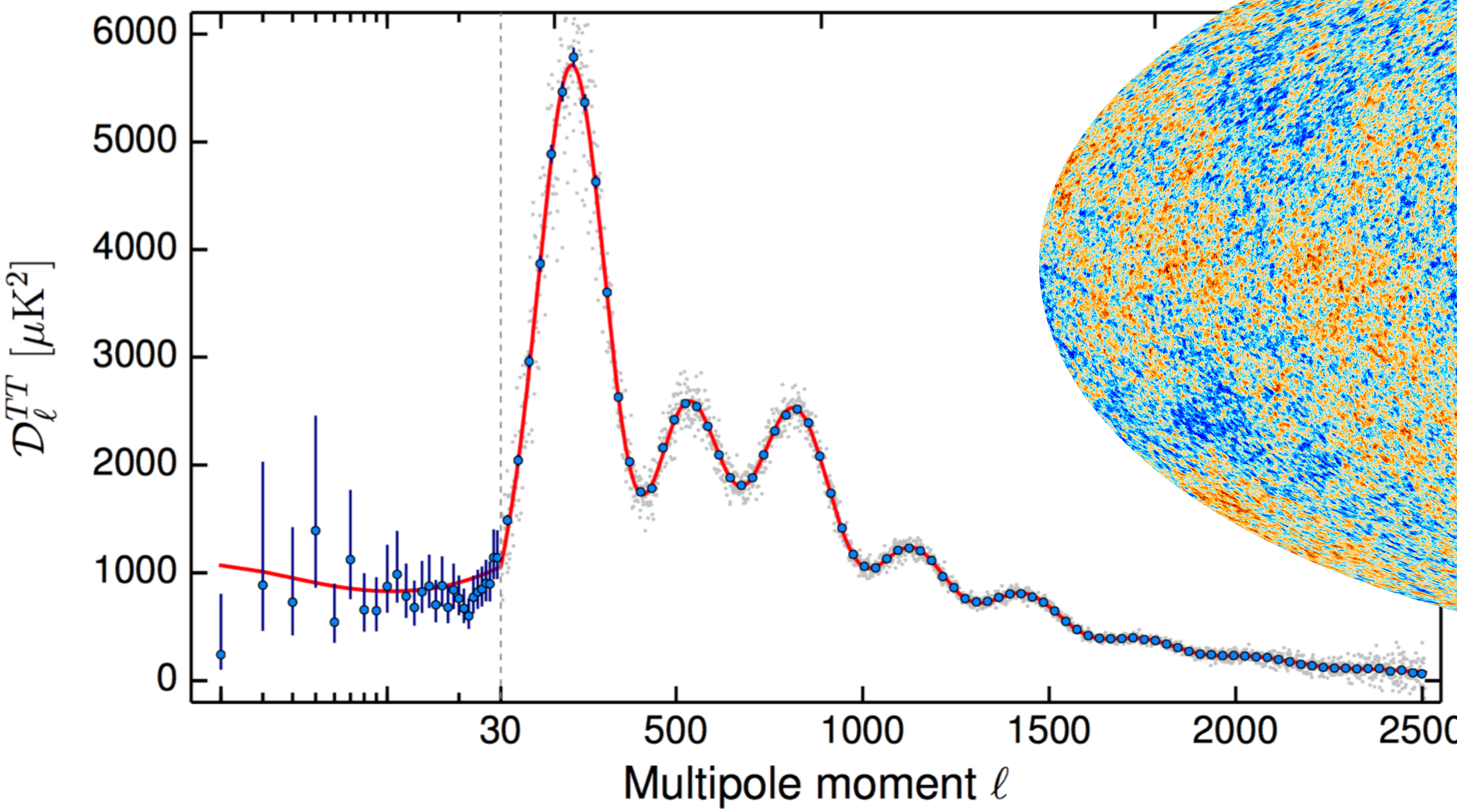

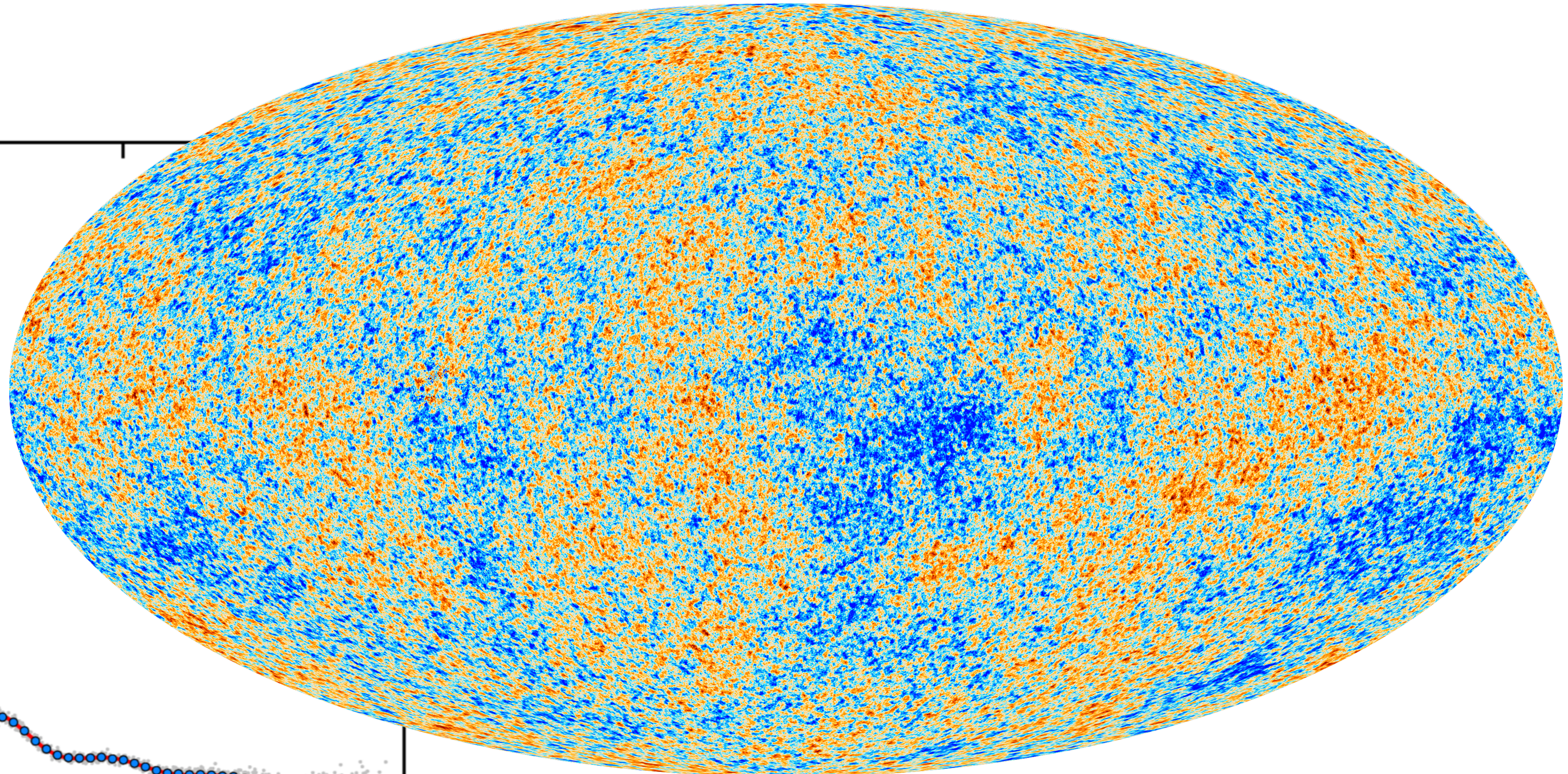

$\Omega_{\mathcal{K}} = 0.000 \pm 0.005$

$n_\mathrm{s} = 0.9639 \pm 0.0047$ almost scale invariant

excluded

$f_\mathrm{NL}^\mathrm{loc} = 0.8 \pm 5$

$f_\mathrm{NL}^\mathrm{eq} = -4 \pm 43$ Gaussian signal

$f_\mathrm{NL}^\mathrm{ort} = -26 \pm 21$ Isocurvature $\lesssim 1\,\%$

$r < 0.08$

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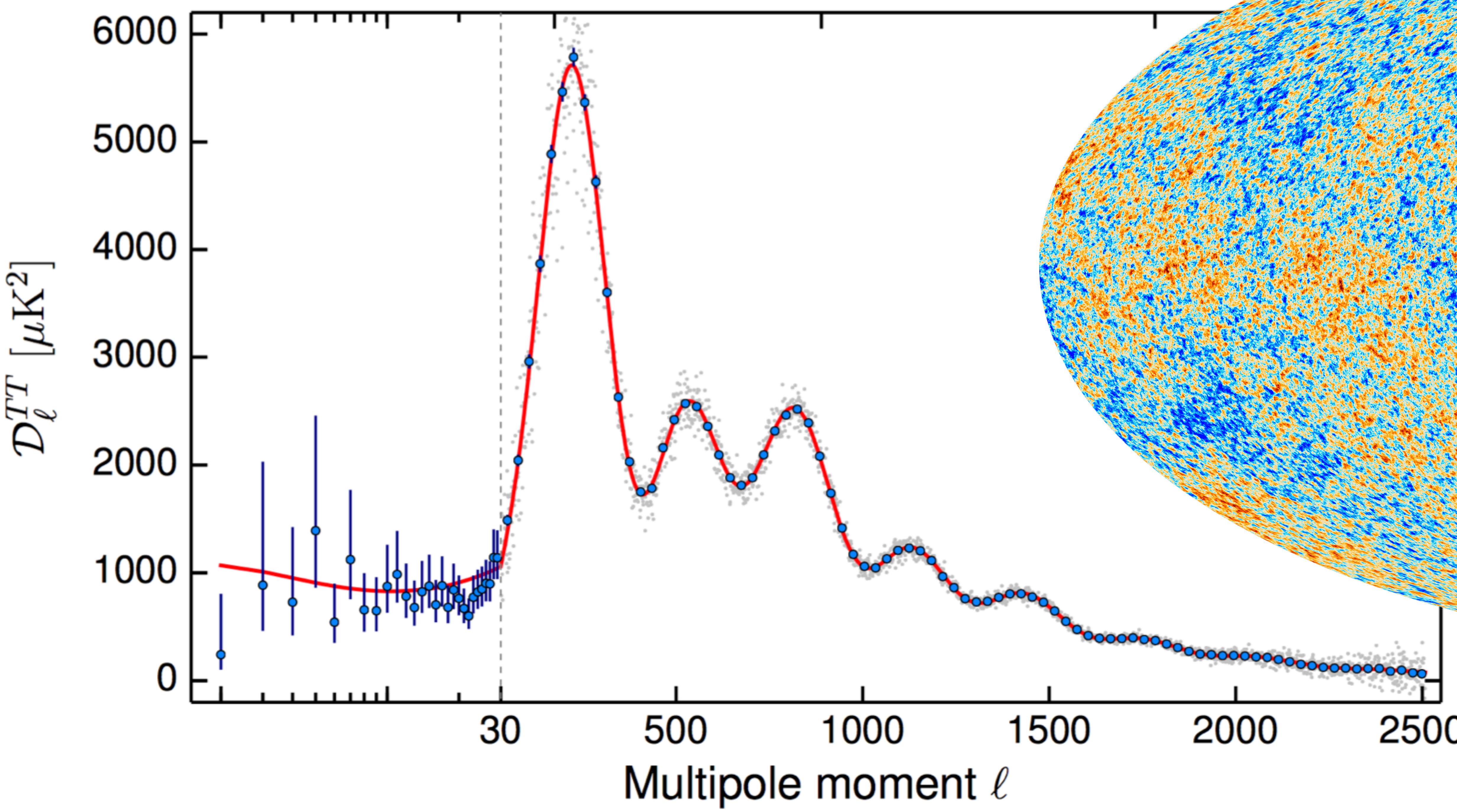

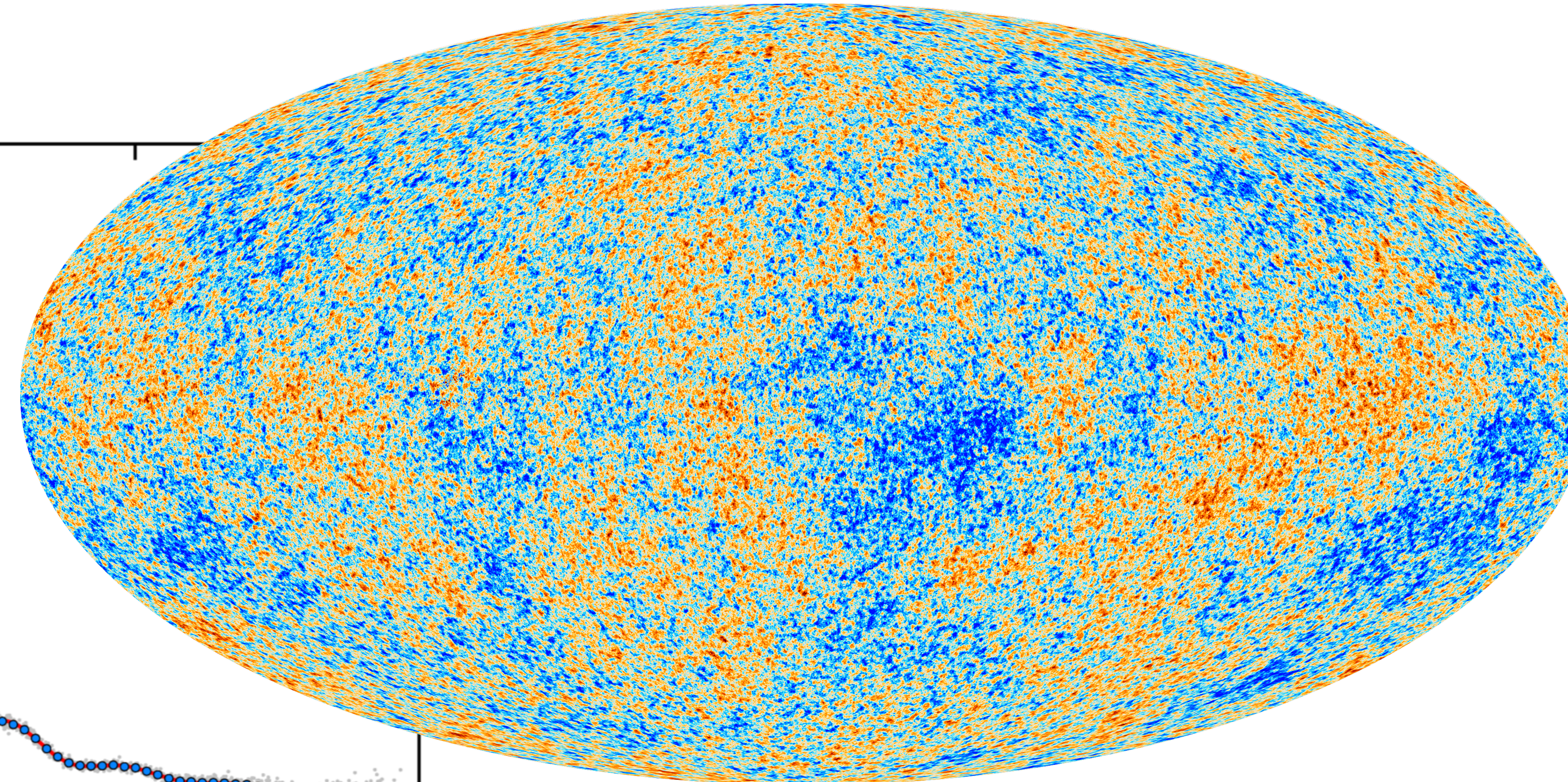

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quantum vacuum fluctuations of a single scalar d.o.f

$r < 0.08$

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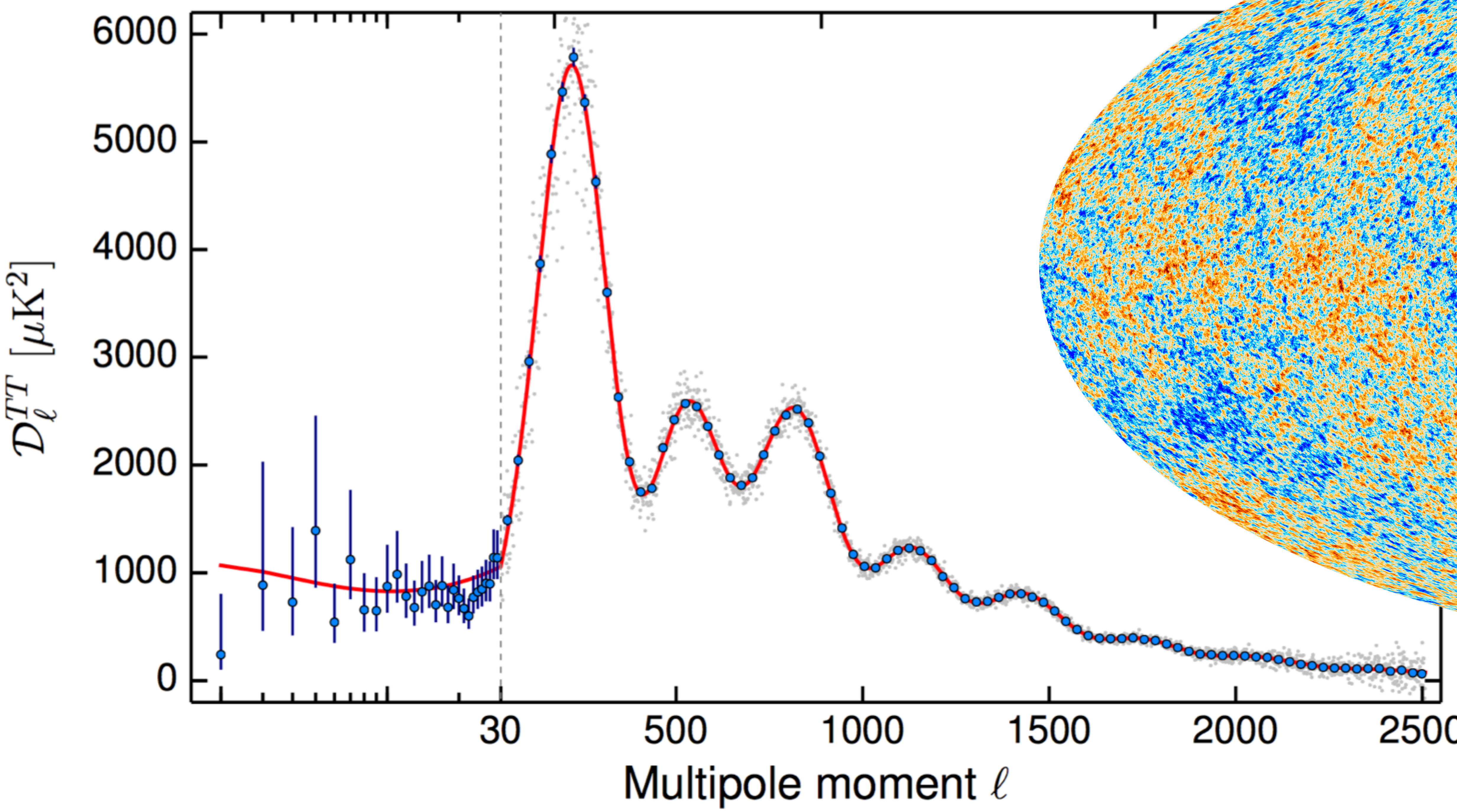

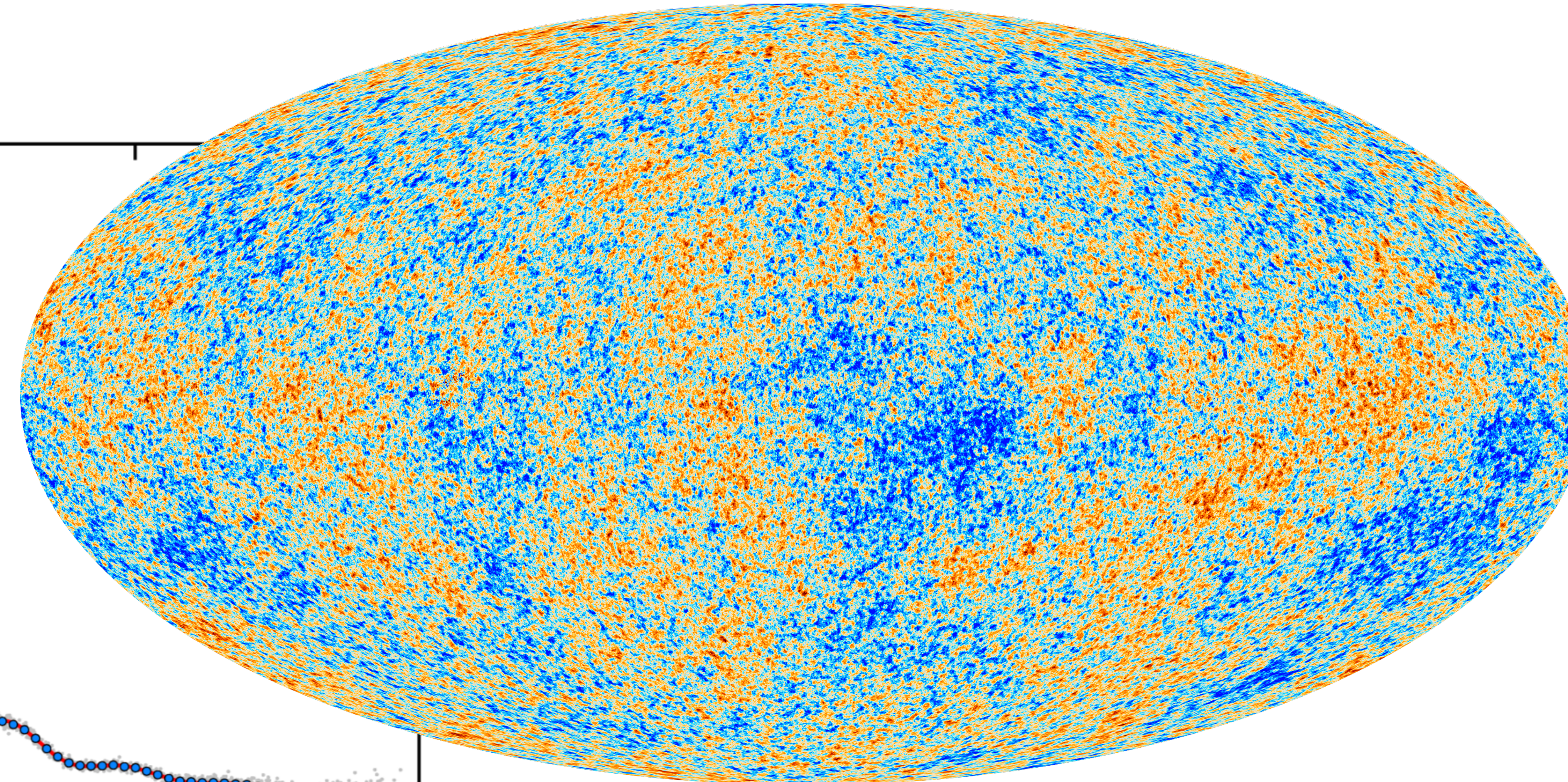

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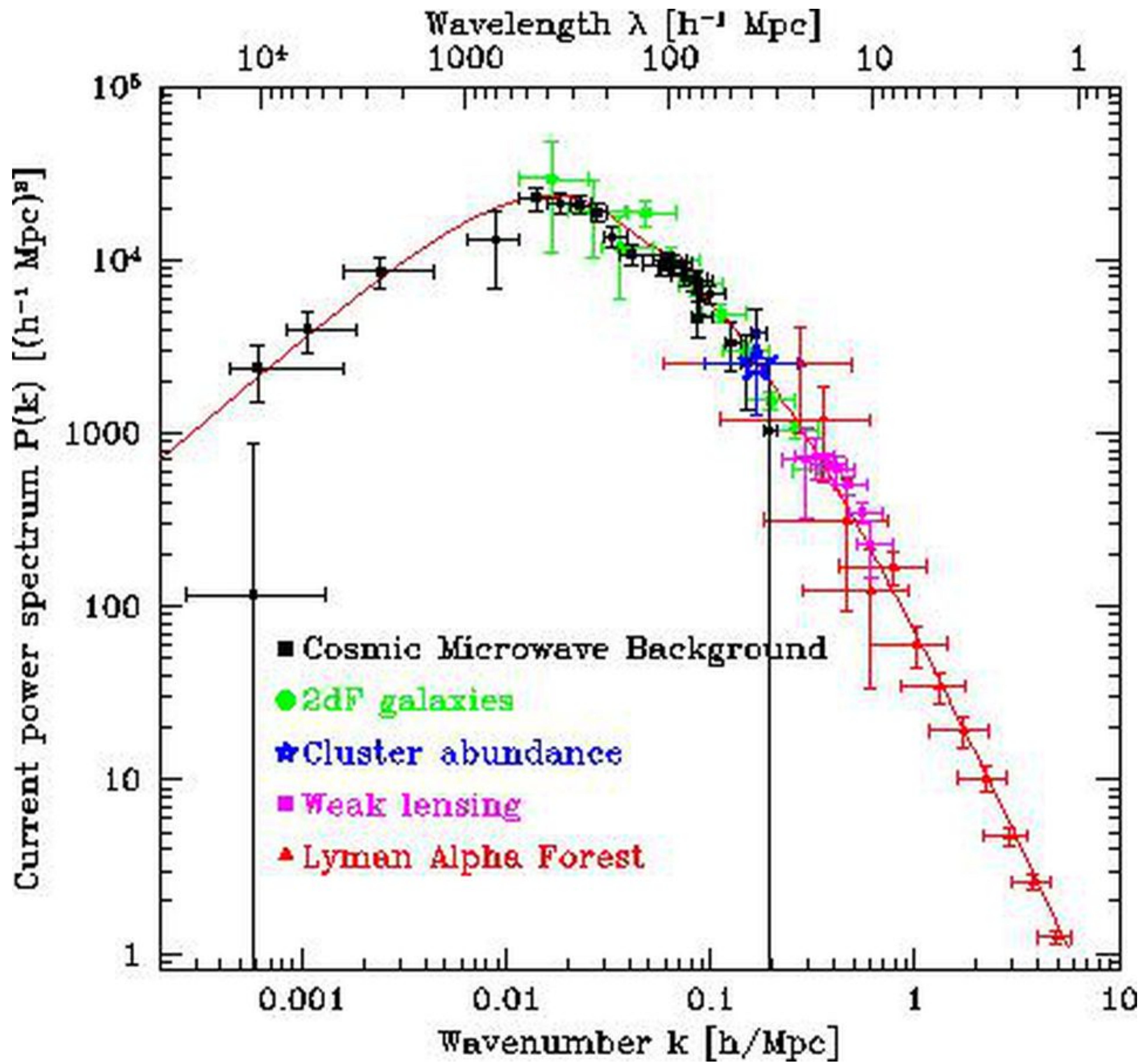
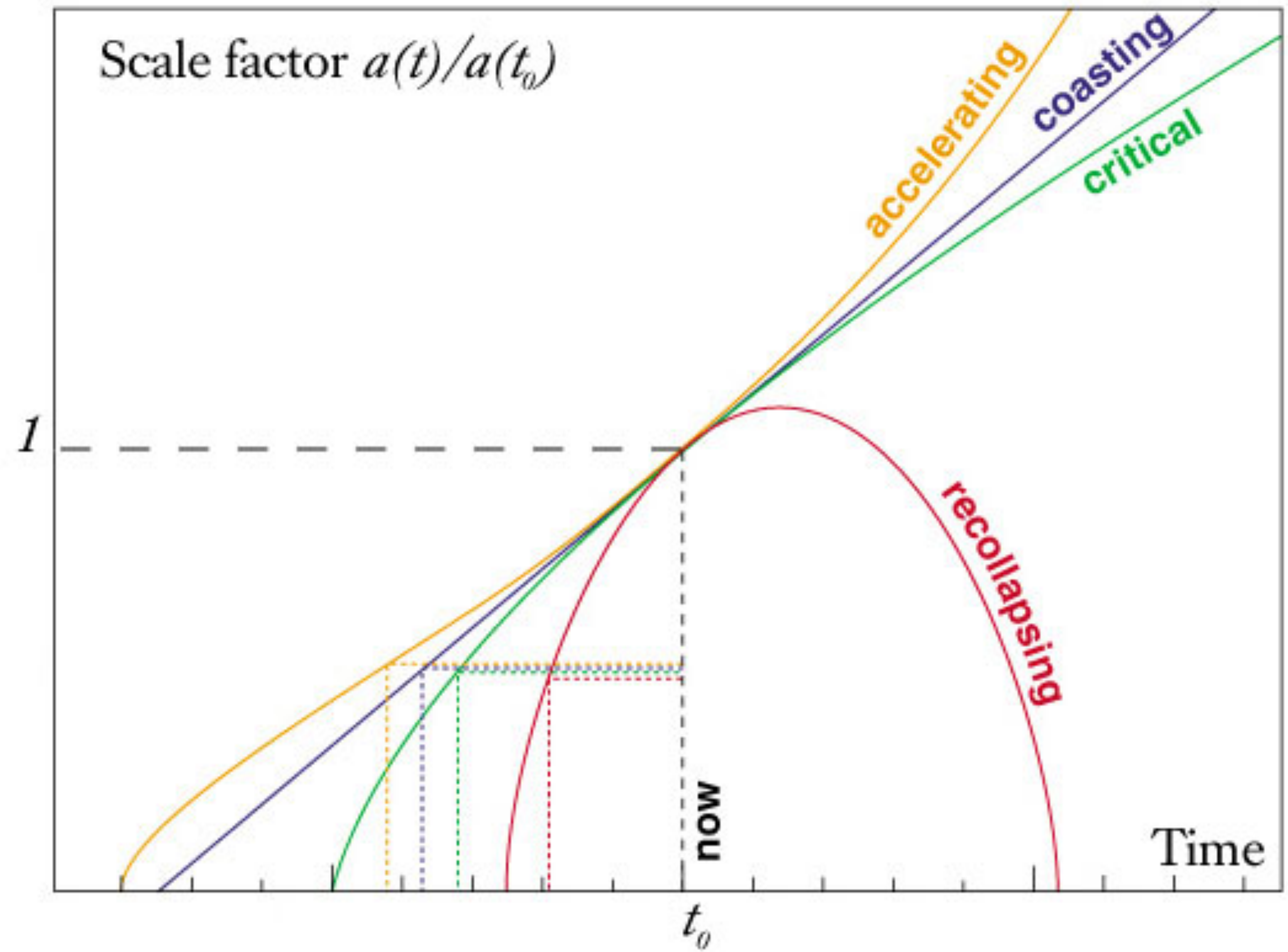
quantum vacuum fluctuations of a single scalar d.o.f

compatible with INFLATION

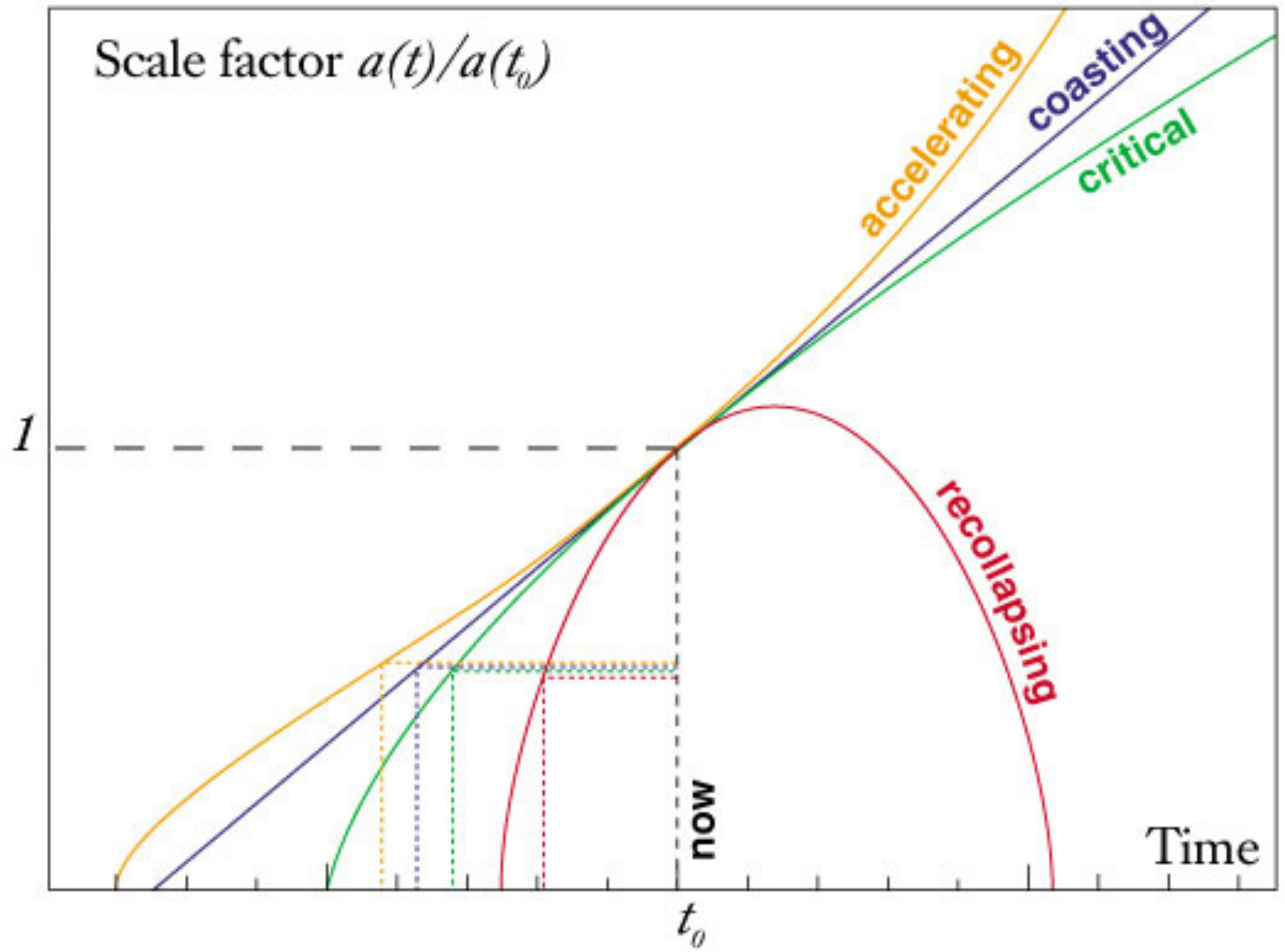
Numerical simulation for
large scale structure formation...



A central problem (though not often formulated thus…): the singularity



Singularity problem… a quantum effect?



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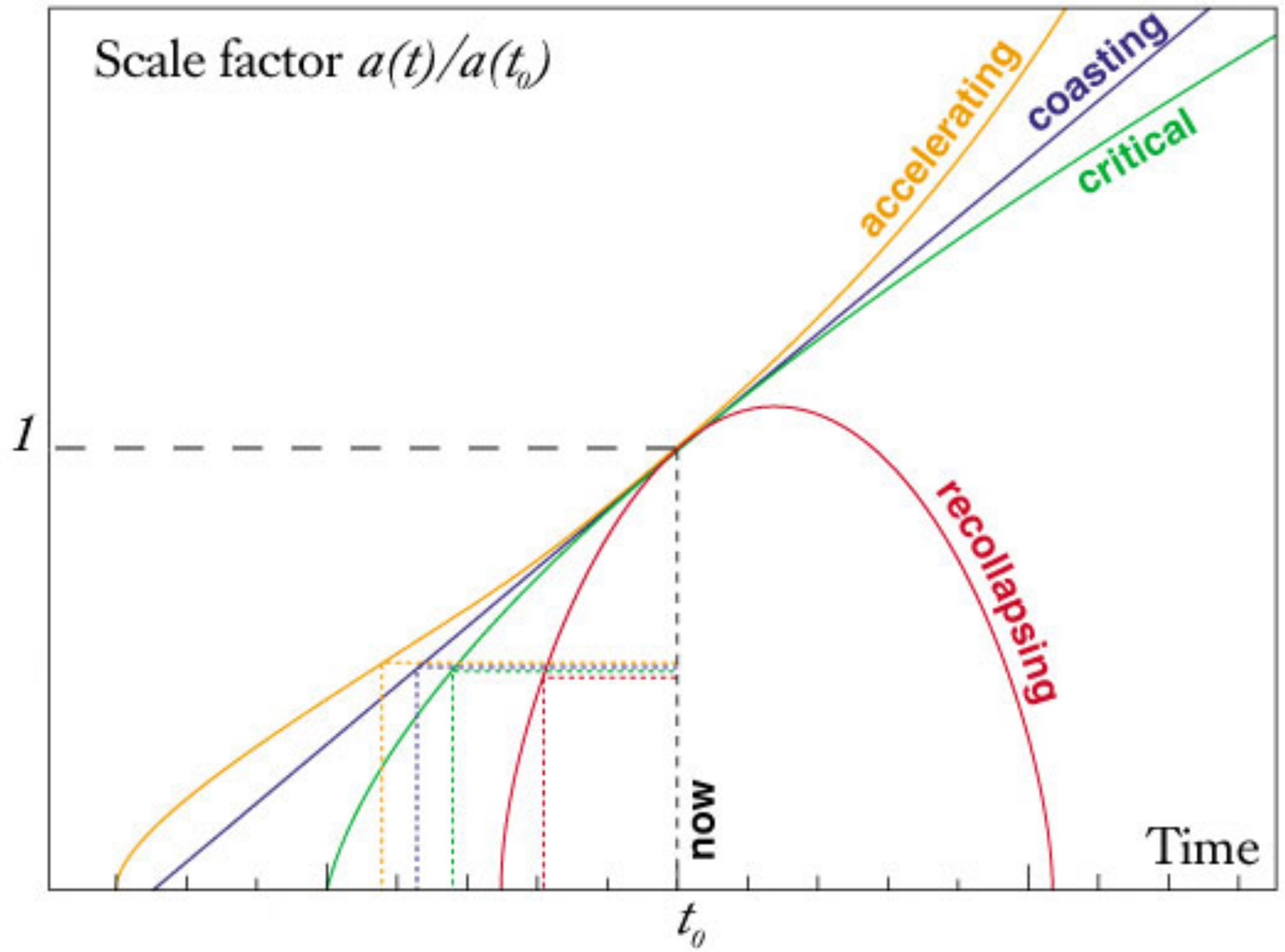
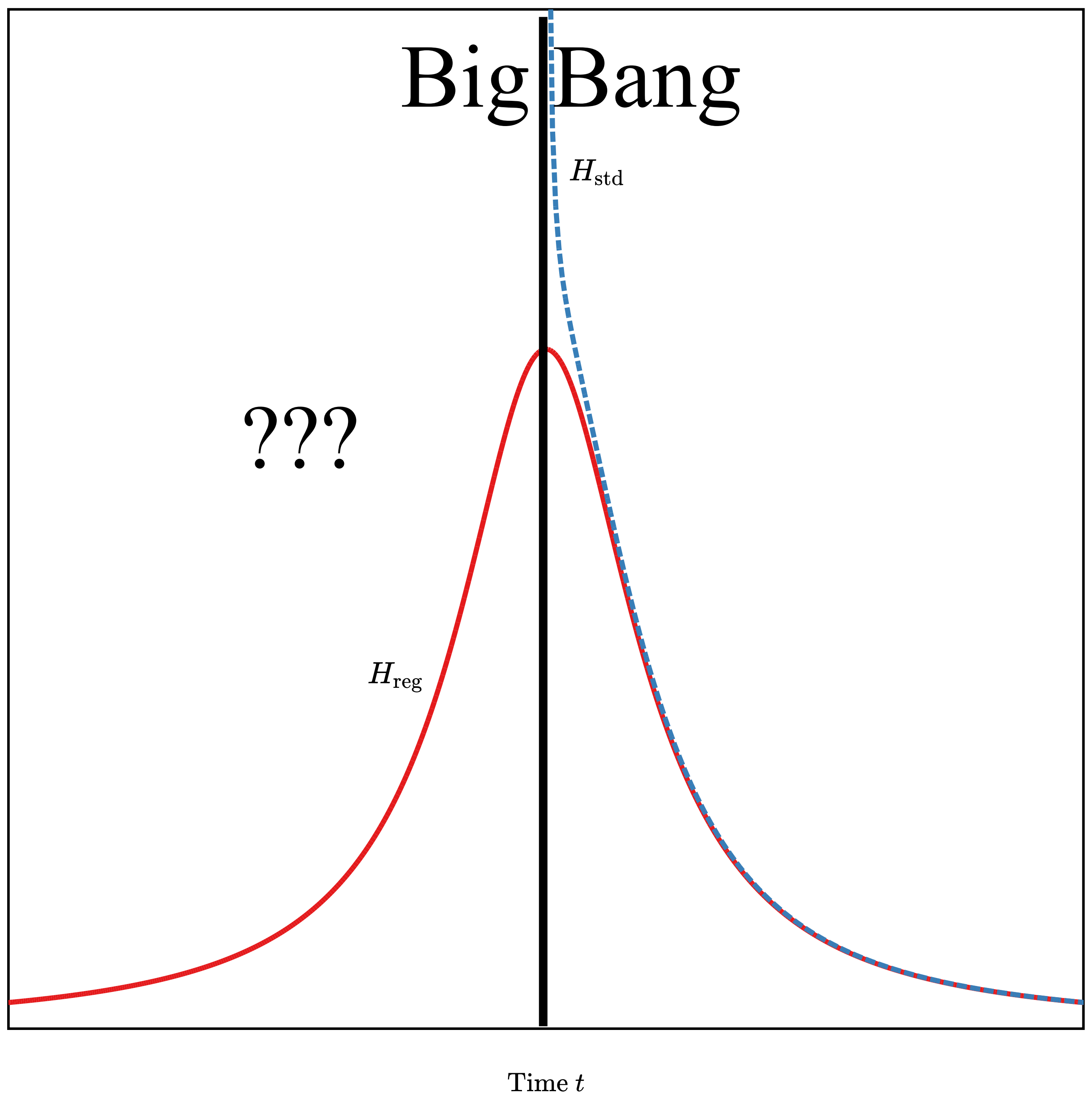
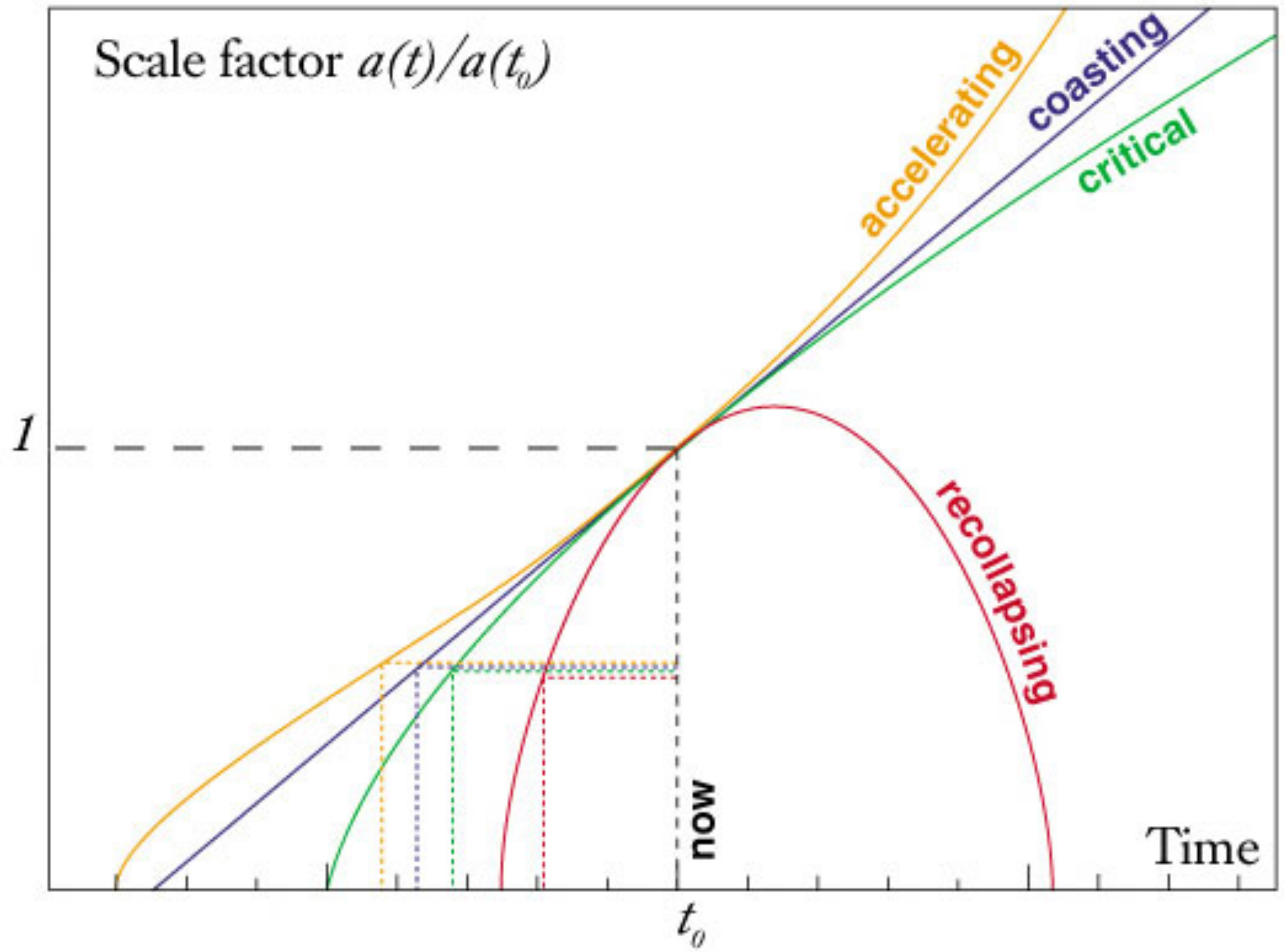
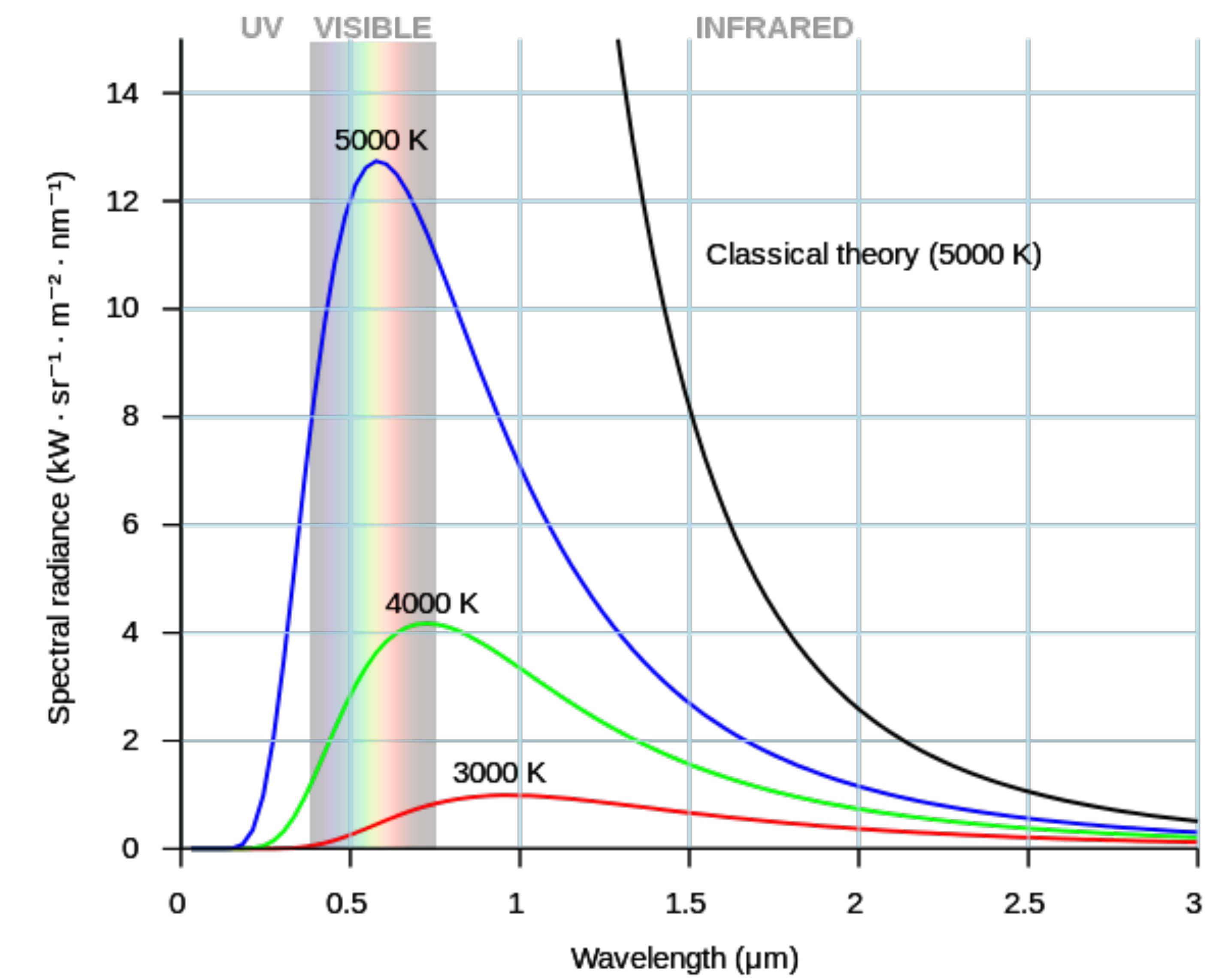


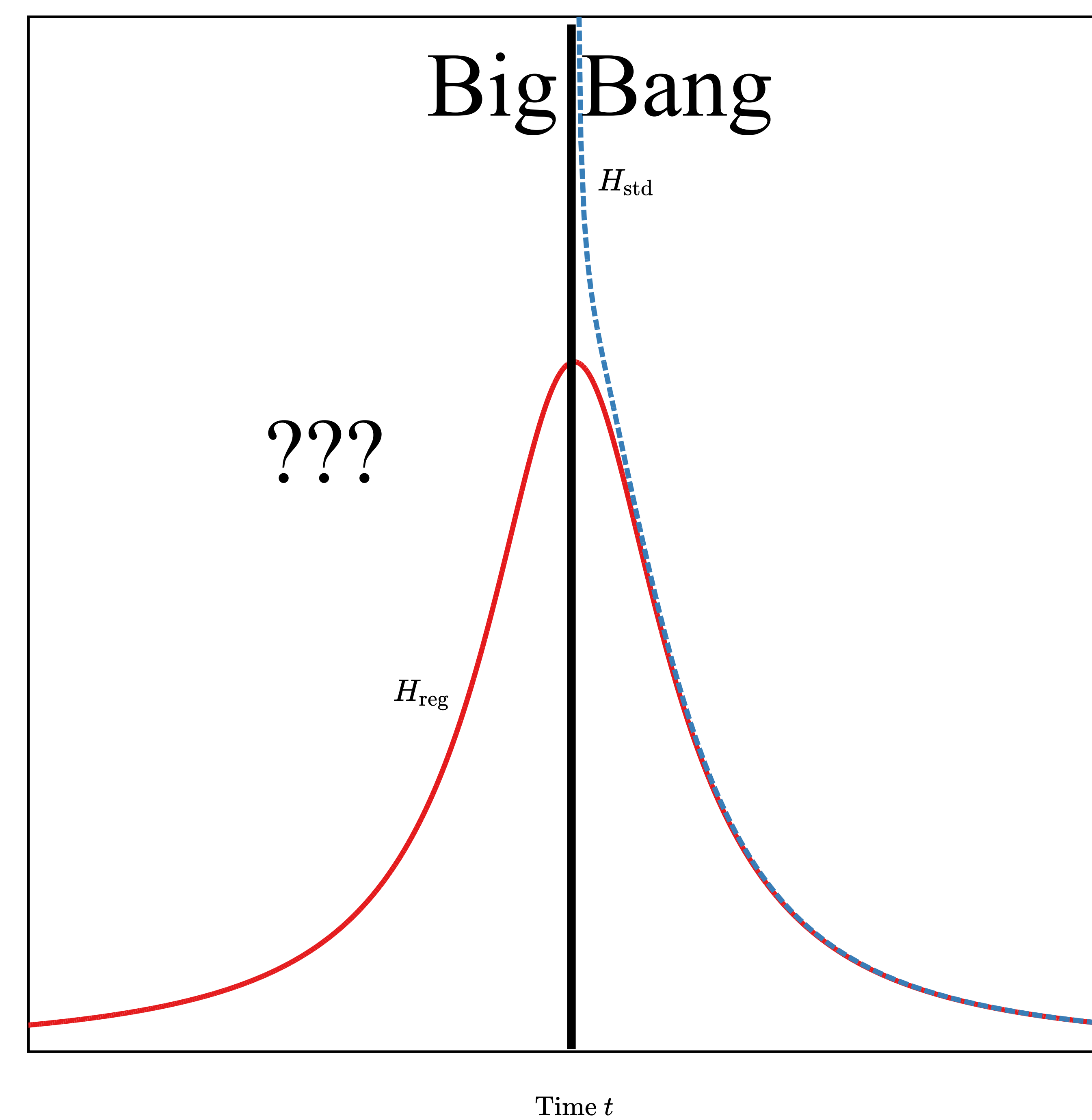


Singularity problem…



a quantum effect?





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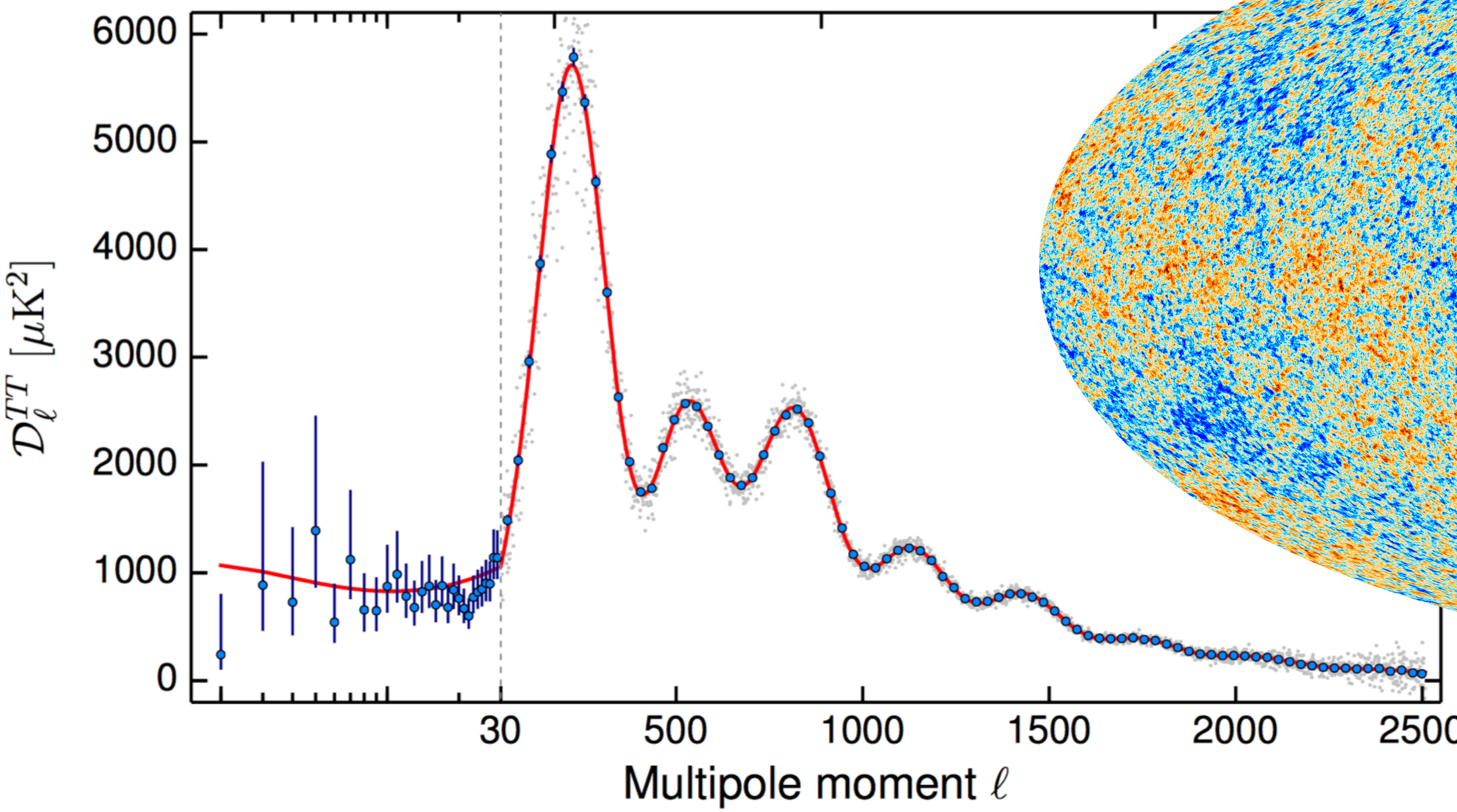

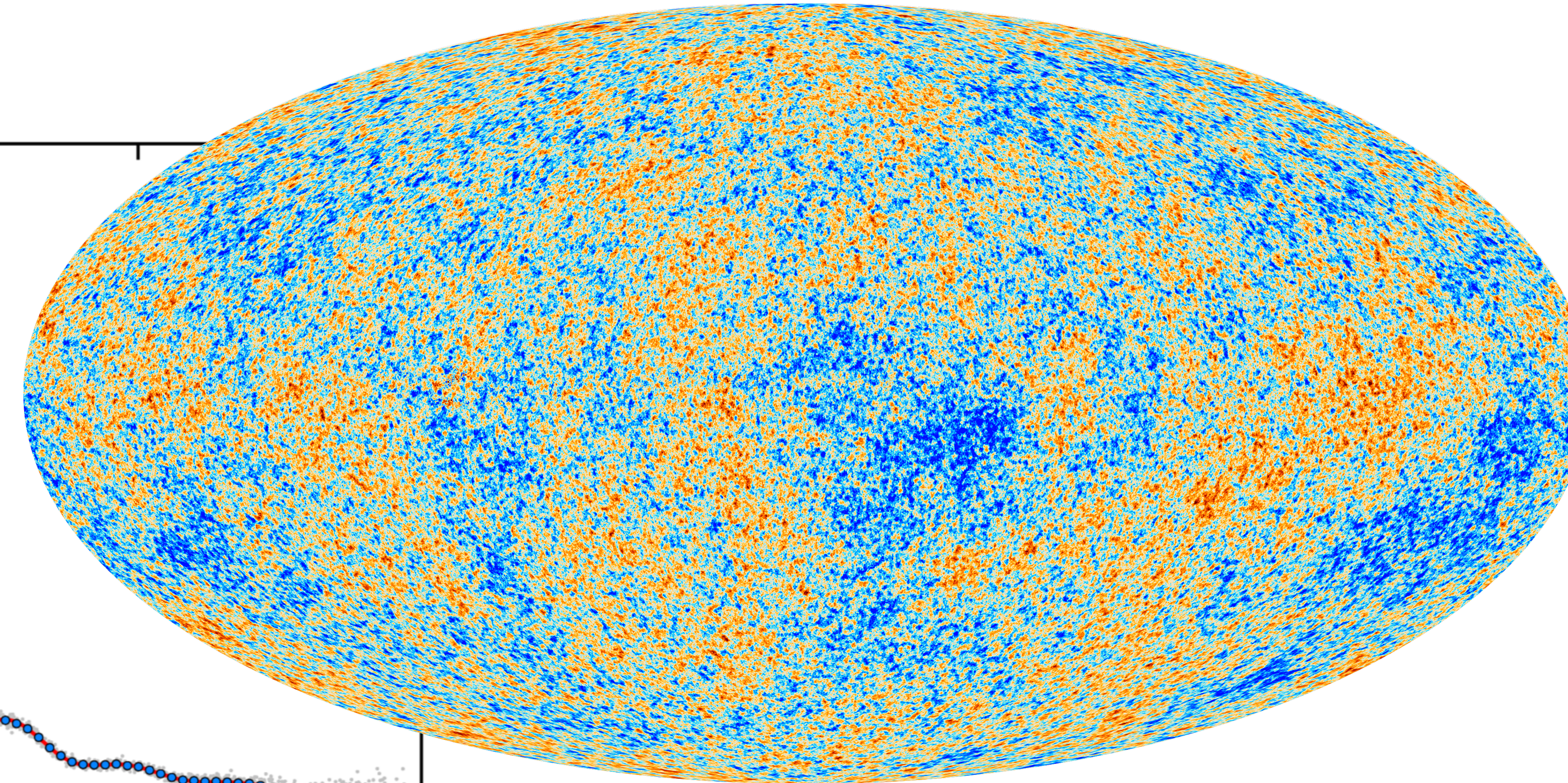

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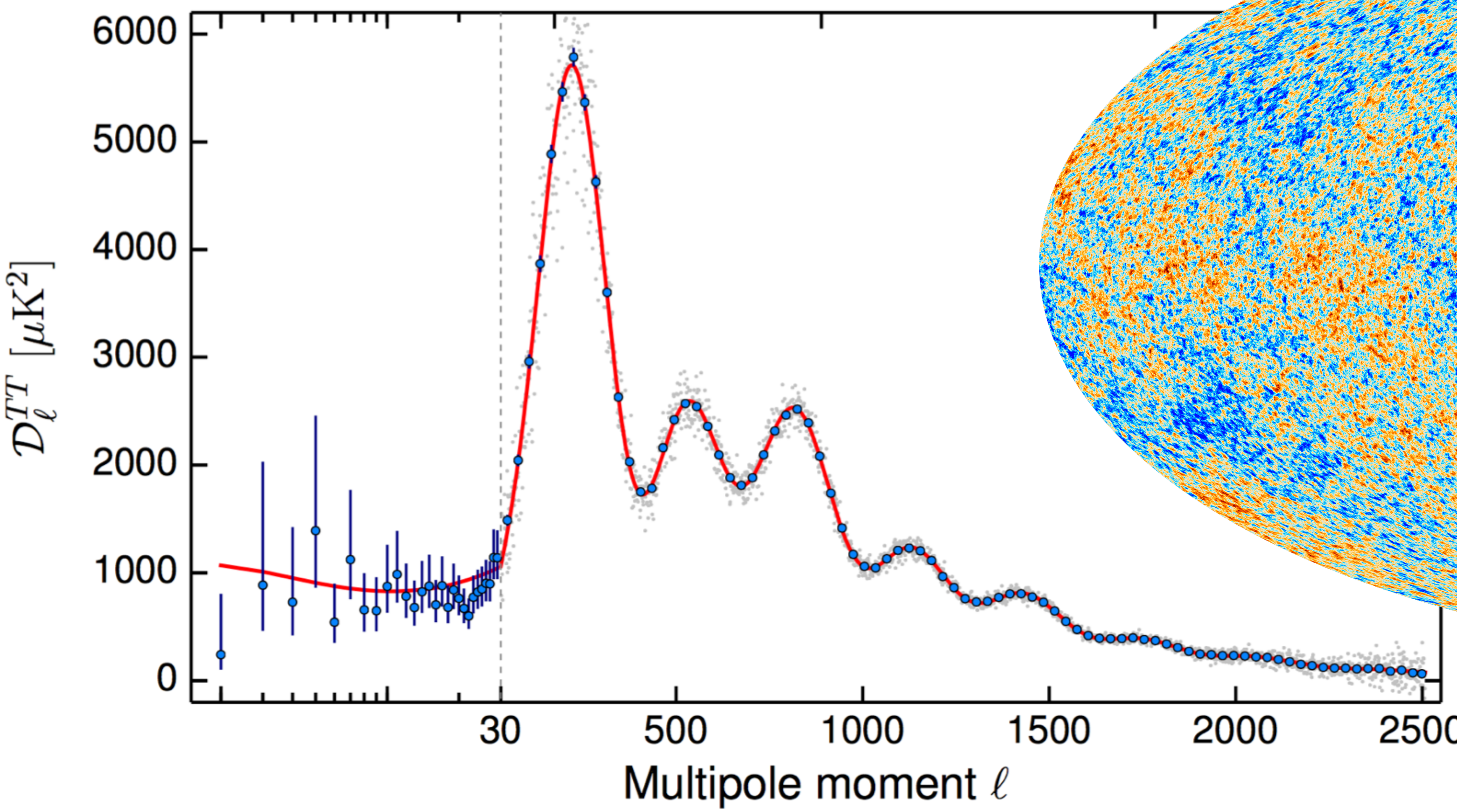

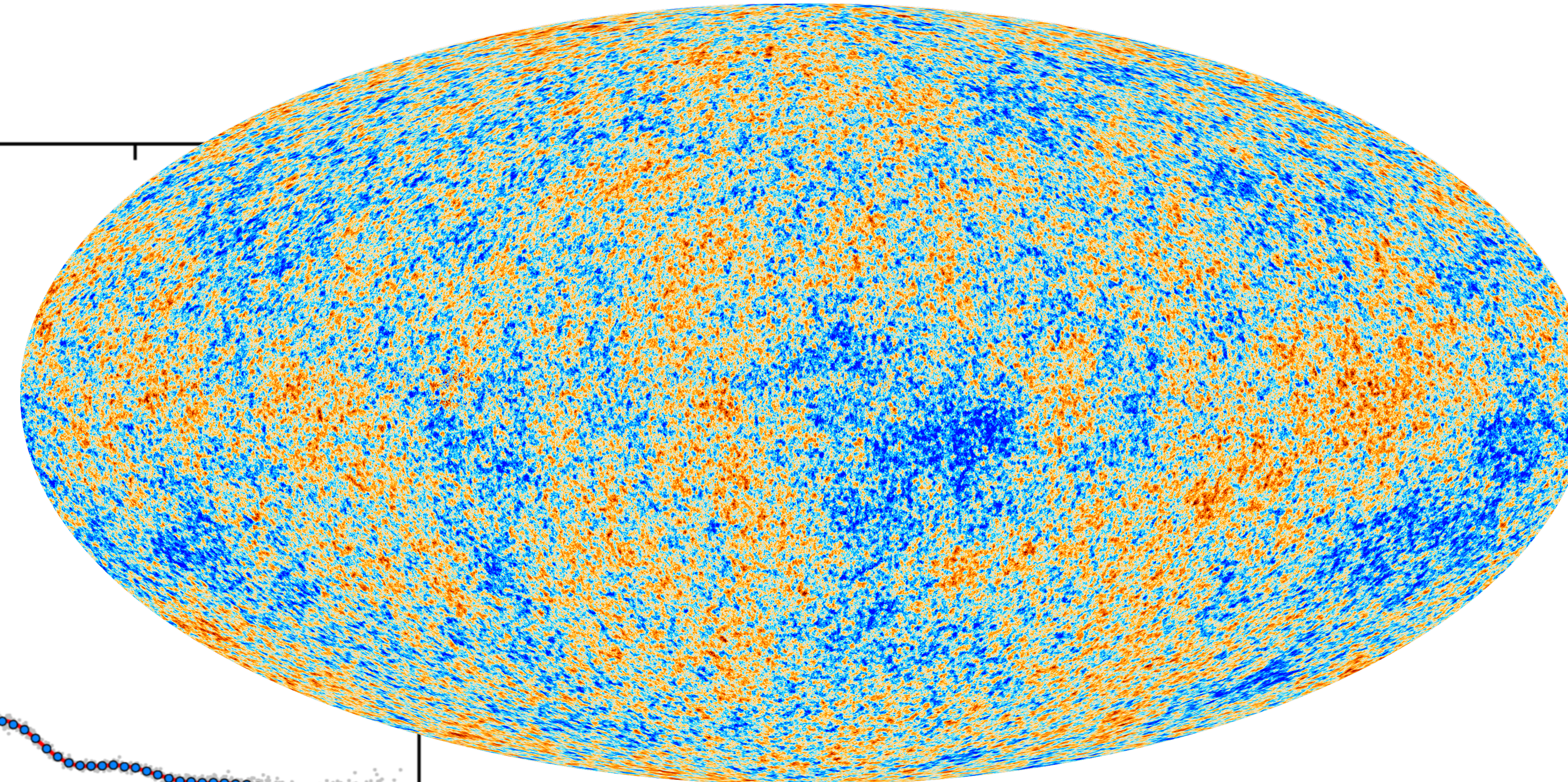

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Quantum mechanics

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $\hat{\mathcal{O}}\,|n\rangle = \omega_n\,|n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}\,|\psi(t)\rangle$

$\hat{H}$ → Hamiltonian

Born rule $\text{Prob}\left[\omega_n; t\right] = |\langle n\,|\,\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

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} Mutually incompatible

+ External observer

Predictions for a quantum theory

Quantum average of observable $\langle \Psi | \hat{\mathcal{O}} | \Psi \rangle$

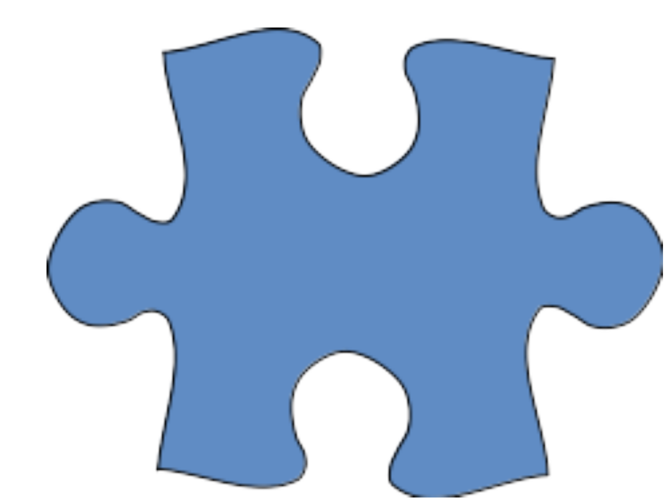
laboratory: repeat experiment

ensemble average ⟷ quantum average

cosmology: a single experiment

→ ergodicity

spatial average (directions in the sky)! ⟷ quantum average



Quantum Cosmology

Hamiltonian GR (3+1)

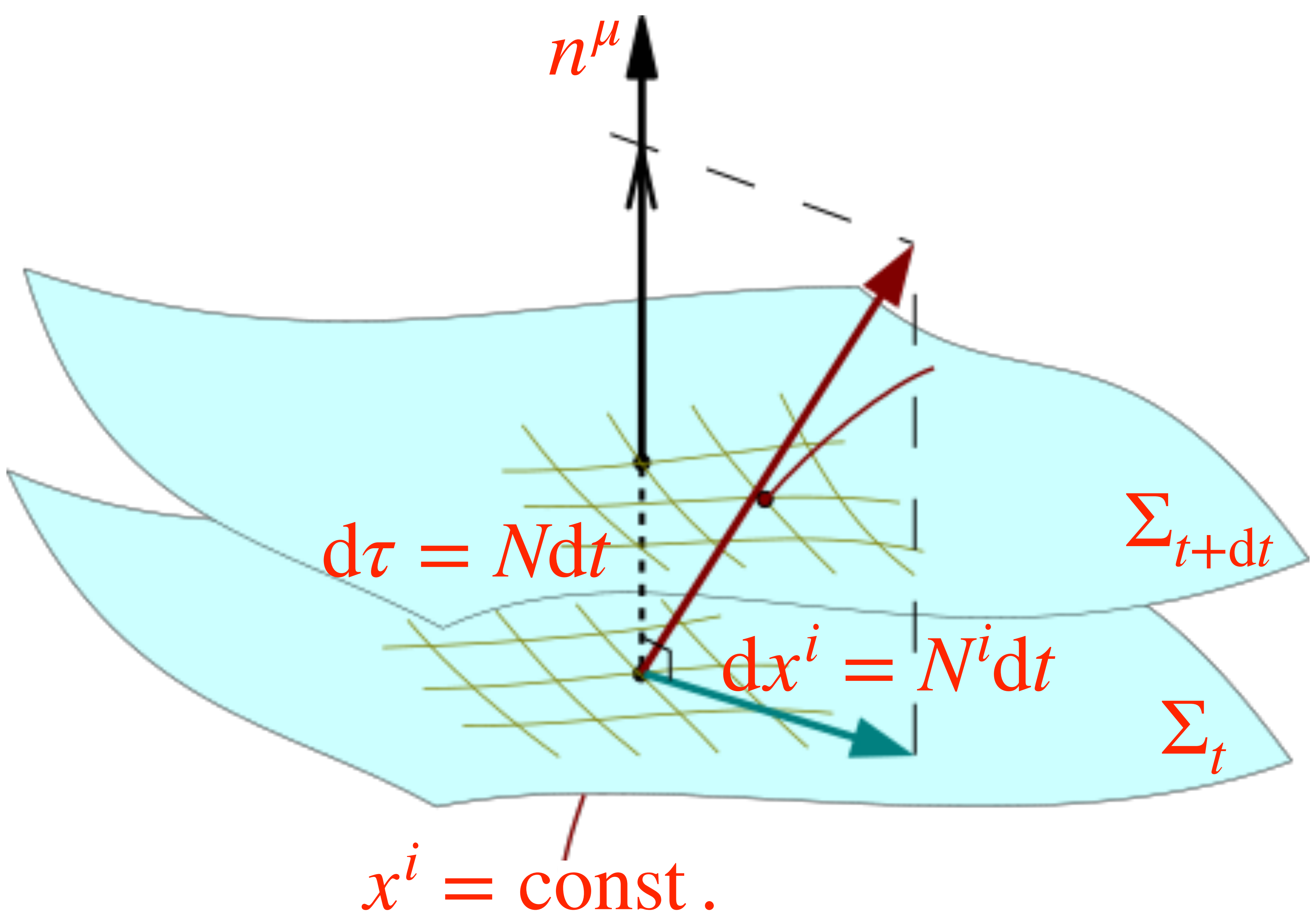
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

lapse function

intrinsic metric =
first fundamental form

shift vector

intrinsic curvature tensor
$^3R^i{}_{jkl}(h)$



extrinsic curvature =
second fundamental form

$$K_{ij} = -\nabla^{(h)}_j n_i = \frac{1}{2N}\left(\nabla^{(h)}_j N_i + \nabla^{(h)}_i N_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action (Einstein-Hilbert, compact space):

$$\mathcal{S} = \frac{1}{16\pi G_{\rm N}}\left[\int_{\mathcal{M}}\sqrt{-g}\ (R-2\Lambda)\,\mathrm{d}^4x + 2\int_{\partial\mathcal{M}}\sqrt{h}K^i{}_i\mathrm{d}^3x\right] + \mathcal{S}_{\rm matter}\left[\Phi(x)\right]$$

$$\longrightarrow \quad \mathcal{S} = \int L\mathrm{d}t = \frac{1}{16\pi G_{\rm N}}\int \mathrm{d}t\left[\int \mathrm{d}^3x\, N\sqrt{h}\left(K_{ij}K^{ij} - K^2 + {}^3R - 2\Lambda\right) + L_{\rm matter}\right]$$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\rm N}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi^{\Phi} \equiv \frac{\delta L}{\delta\dot{\Phi}} = -\frac{\sqrt{h}}{N}\left(\dot{\Phi} - N\frac{\partial\Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{N}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{N}^i} = 0 \end{aligned}\right\} \quad \text{primary constraints}$$

Hamiltonian $$H \equiv \int \mathrm{d}^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + \pi^{ij} \dot{h}_{ij} + \pi^\Phi \dot{\Phi}\right) - L$$

$$= \int \mathrm{d}^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + N\mathscr{H} + N_i \mathscr{H}^i\right)$$

variation wrt lapse: $\mathscr{H} = 0$ ⟶ Hamiltonian constraint

variation wrt shift: $\mathscr{H}^i = 0$ ⟶ momentum constraint

$\Longrightarrow$ complete classical description

Superspace & canonical quantisation

relevant configuration space $\quad \mathrm{Riem}\,(\Sigma) \equiv \left\{ h_{ij}\,(x^\mu), \Phi\,(x^\mu)\,|\,x \in \Sigma \right\}$

$$\mathrm{GR} \Longrightarrow \text{invariance/diffeomorphisms} \Longrightarrow \mathrm{Conf} = \frac{\mathrm{Riem}\,(\Sigma)}{\mathrm{Diff}\,(\Sigma)} : \text{Superspace}$$

wave functional $\quad \Psi \left[h_{ij}\,(x), \Phi\,(x) \right]$

+ Dirac canonical quantisation procedure

$$\pi^{ij} \to -i \frac{\delta}{\delta h_{ij}} \qquad \pi^{\Phi} \to -i \frac{\delta}{\delta \Phi} \qquad \pi^{0} \to -i \frac{\delta}{\delta N} \qquad \pi^{i} \to -i \frac{\delta}{\delta N_i}$$

primary constraints
$$\left\{\begin{aligned}\hat{\pi}^0 &= -i\frac{\delta\Psi}{\delta N} = 0\\ \hat{\pi}^i &= -i\frac{\delta\Psi}{\delta N_i} = 0\end{aligned}\right.$$

momentum $\hat{\mathrm{H}}^i\Psi = 0 \quad \Longrightarrow \quad i\,\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\mathrm{N}}\hat{T}^{0i}\Psi$

same Ψ for configurations related by a coordinate transformation

Hamiltonian
$$\hat{\mathscr{H}}\Psi = \left[-16\pi G_{\mathrm{N}}\mathscr{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(-{}^3R + 2\Lambda + 16\pi G_{\mathrm{N}}\hat{T}^{00}\right)\right]\Psi = 0$$

Wheeler-De Witt equation

$$\mathscr{G}_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

De Witt metric

primary constraints
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Hamiltonian

$$\hat{H}\Psi = 0$$

Wheeler-De Witt equation

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

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Actually make calculations!

Exemple : Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

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Perfect fluid: Schutz formalism ('70)

$$p = p_0\left[\frac{\dot{\varphi}+\theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi,\theta,s) =$ Velocity potentials

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canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ …

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$N \rightarrow a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

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$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}}$$

space defined by $\chi > 0$ ⟶ constraint $\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$

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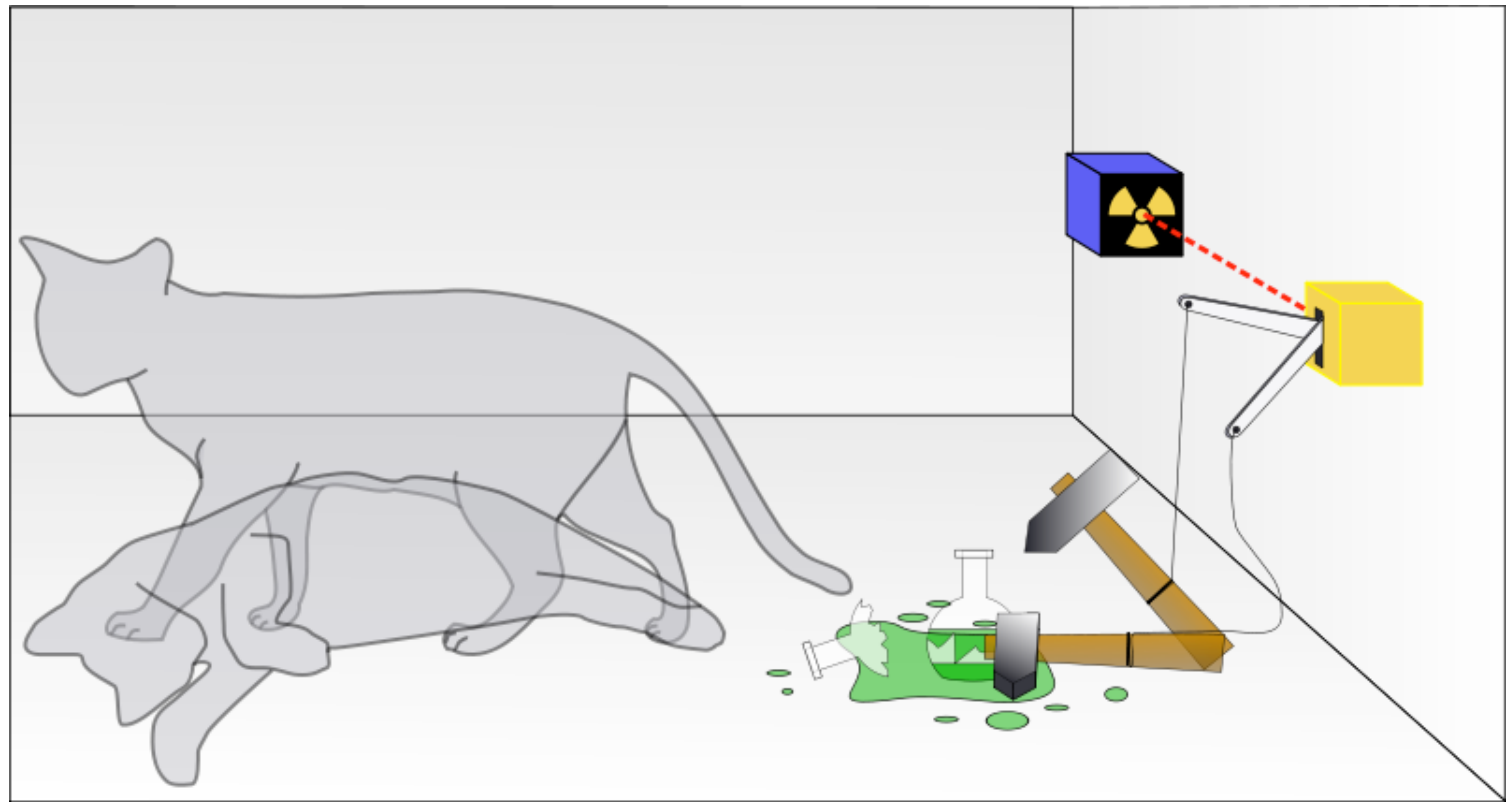
Gaussian wave packet

$$\Rightarrow \Psi = \left[\frac{8T_0}{\pi\left(T_0^2 + T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0 \chi^2}{T_0^2 + T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $S = \dfrac{T\chi^2}{T_0^2 + T^2} + \dfrac{1}{2}\arctan\dfrac{T_0}{T} - \dfrac{\pi}{4}$

What do we do with the wave function of the Universe???

The measurement problem in quantum mechanics

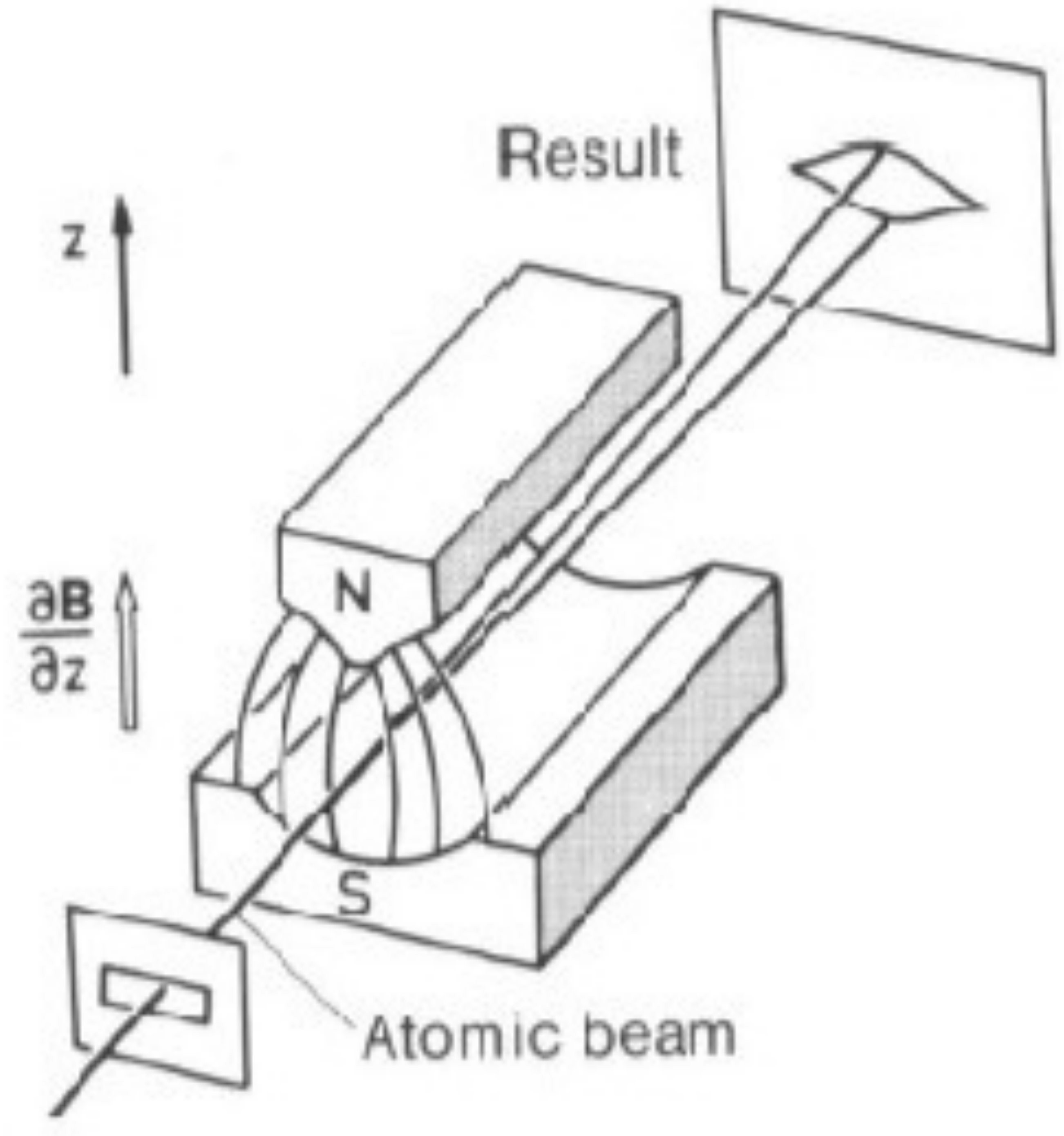


Preferred basis: no unique definition of measured observables

Definite outcome: we don't measure superpositions

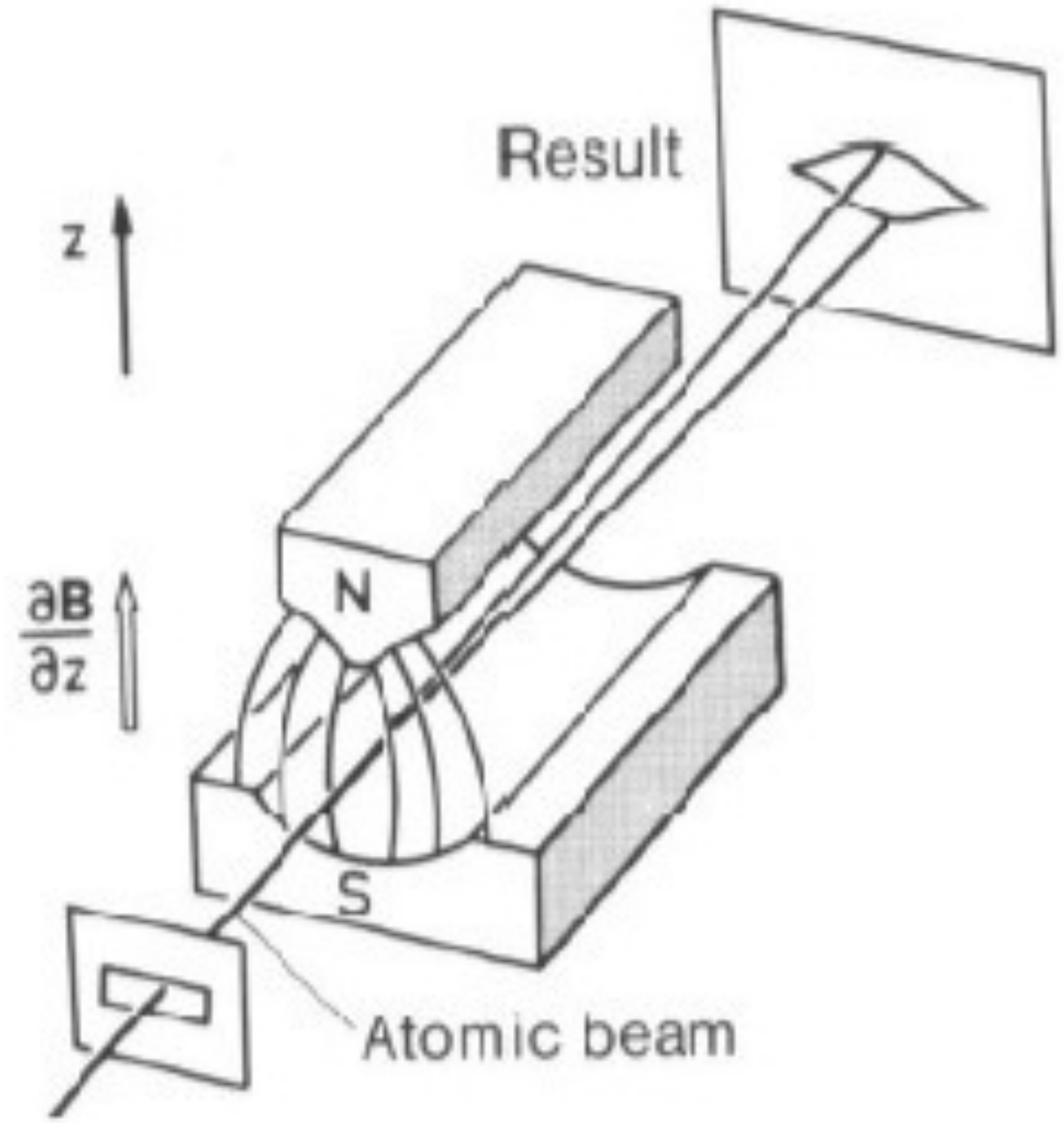
→ collapse of the wave function

The measurement problem in quantum mechanics



Stern-Gerlach

The measurement problem in quantum mechanics



Statistical mixture

$$\Big\{ |\uparrow\rangle \otimes |SG_{\uparrow}\rangle \Big\} \cup \Big\{ |\downarrow\rangle \otimes |SG_{\downarrow}\rangle \Big\}$$

Stern-Gerlach

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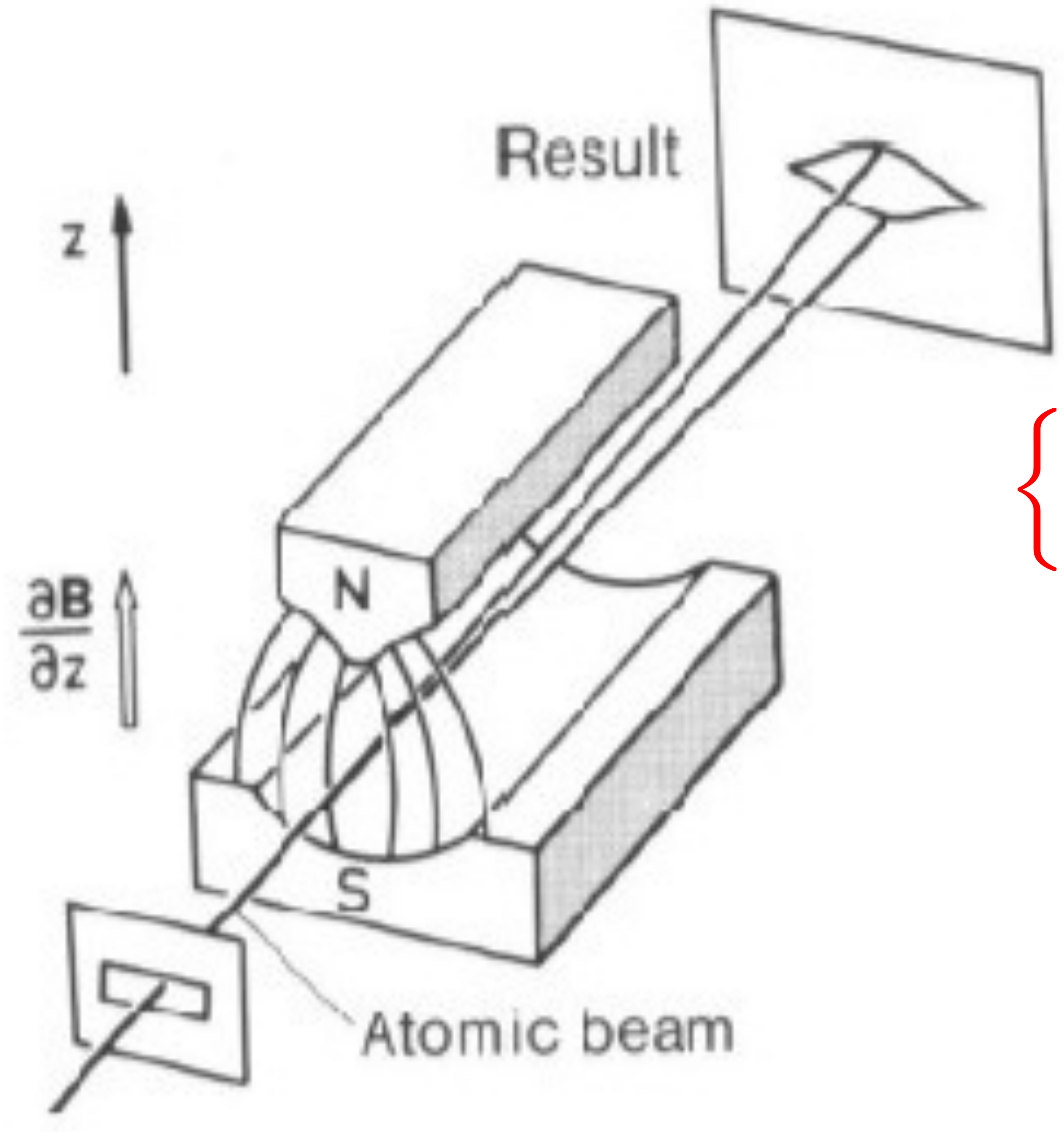

Stern-Gerlach

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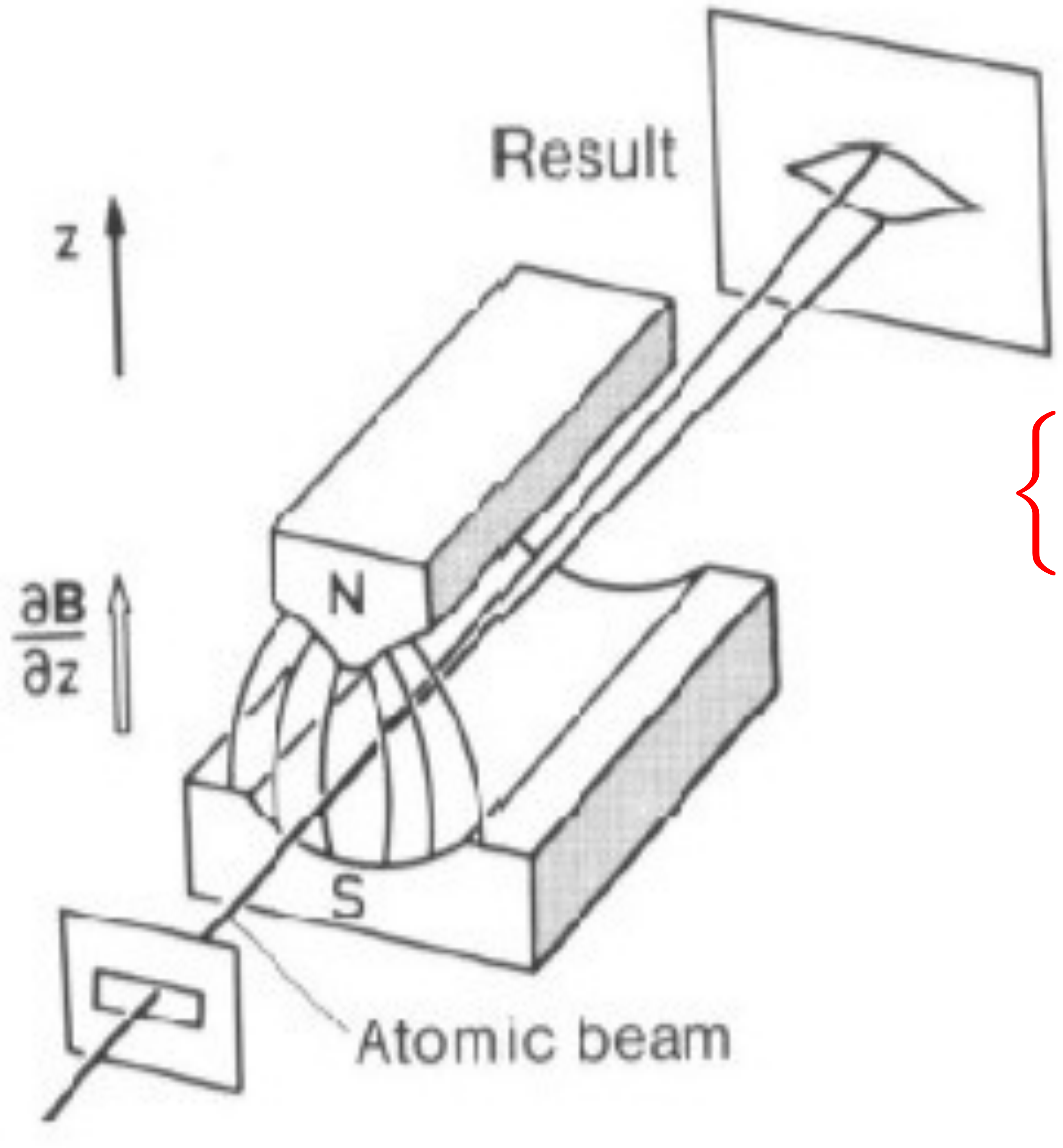
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_\uparrow\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_\downarrow\rangle\right\}$$

$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

$$\left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

The measurement problem in quantum mechanics



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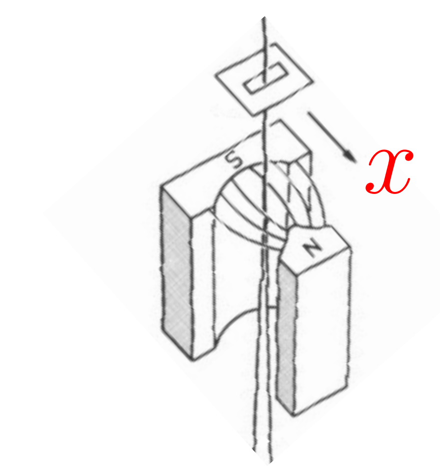
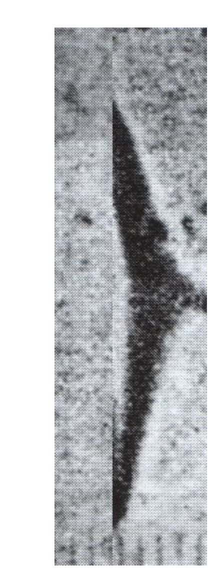
$$\left\{ (|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\}$$





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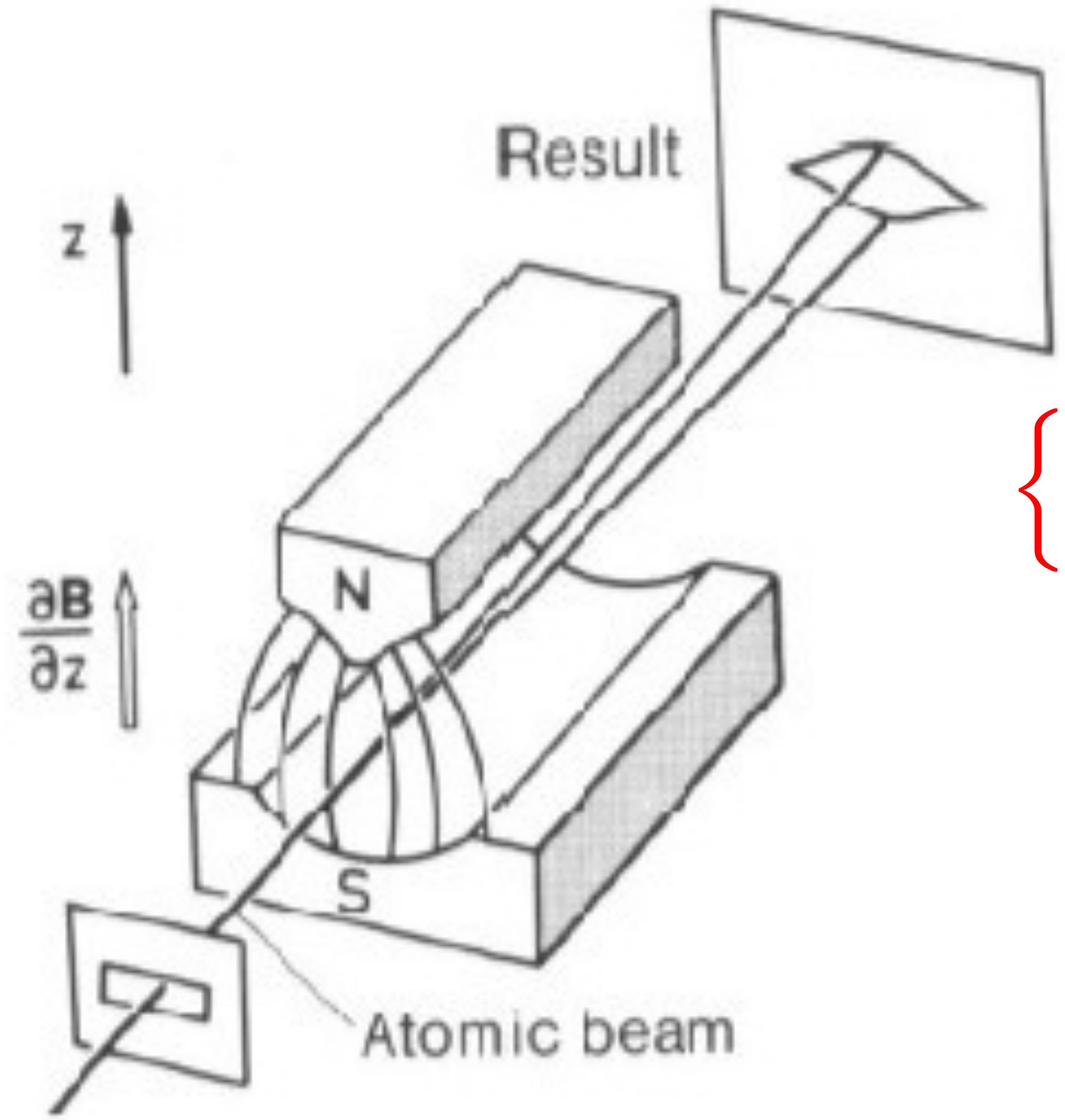

Stern-Gerlach

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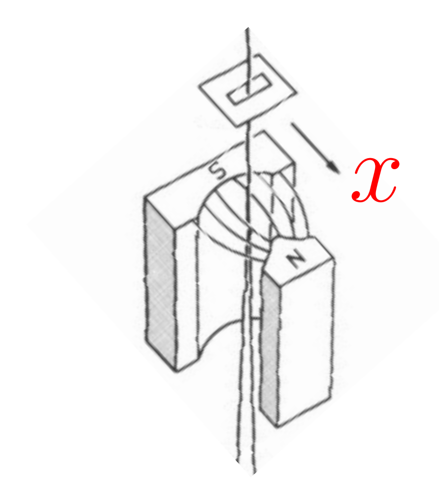
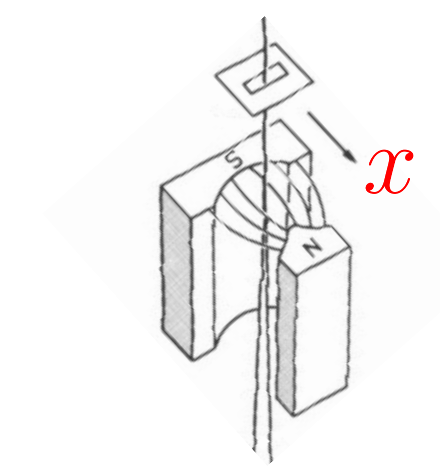
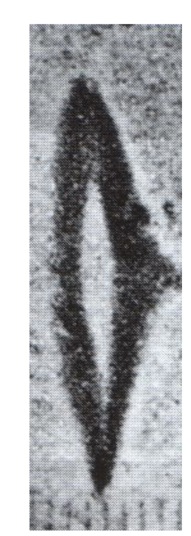
$$\left\{ |\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle \right\} \cup \left\{ |\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle \right\}$$

$$\left\{ |\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\} \cup \left\{ |\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\}$$

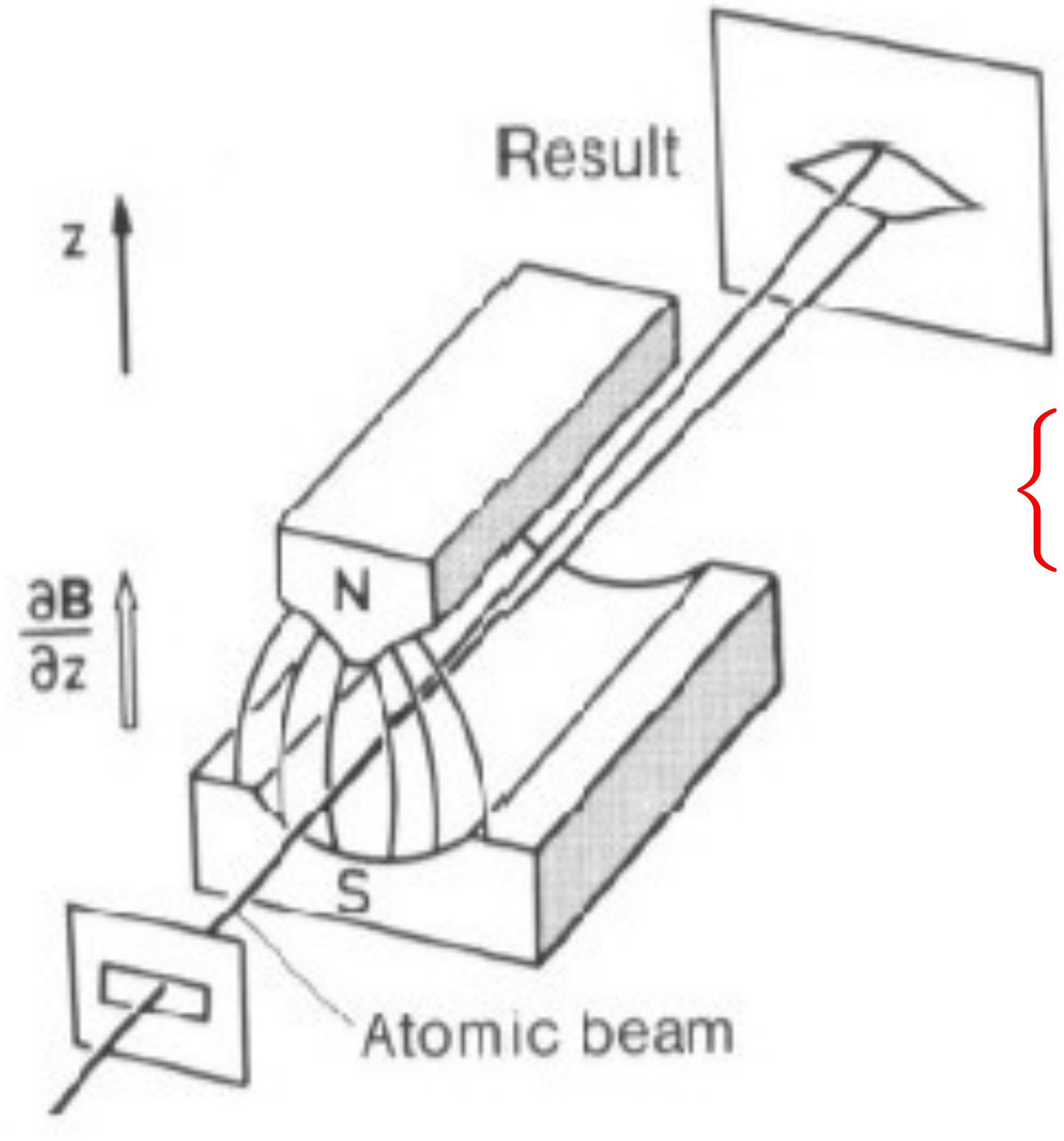
$$\left\{ (|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle \right\}$$







The measurement problem in quantum mechanics



Stern-Gerlach

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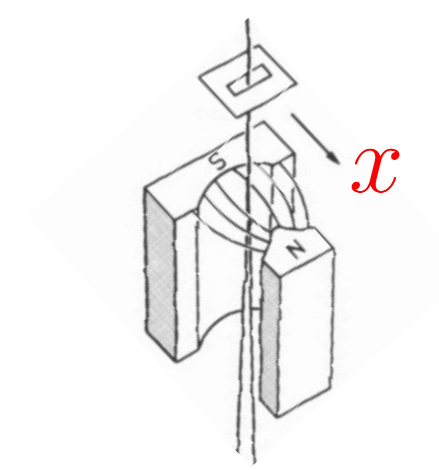
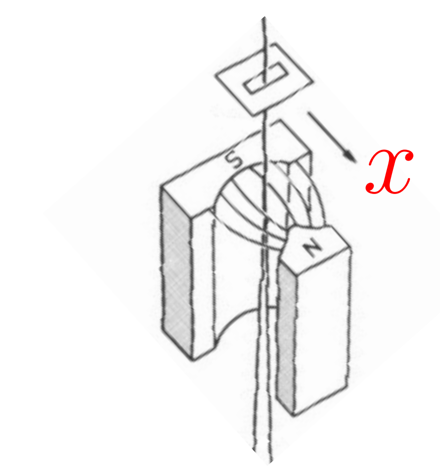
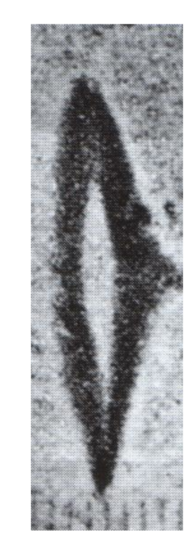
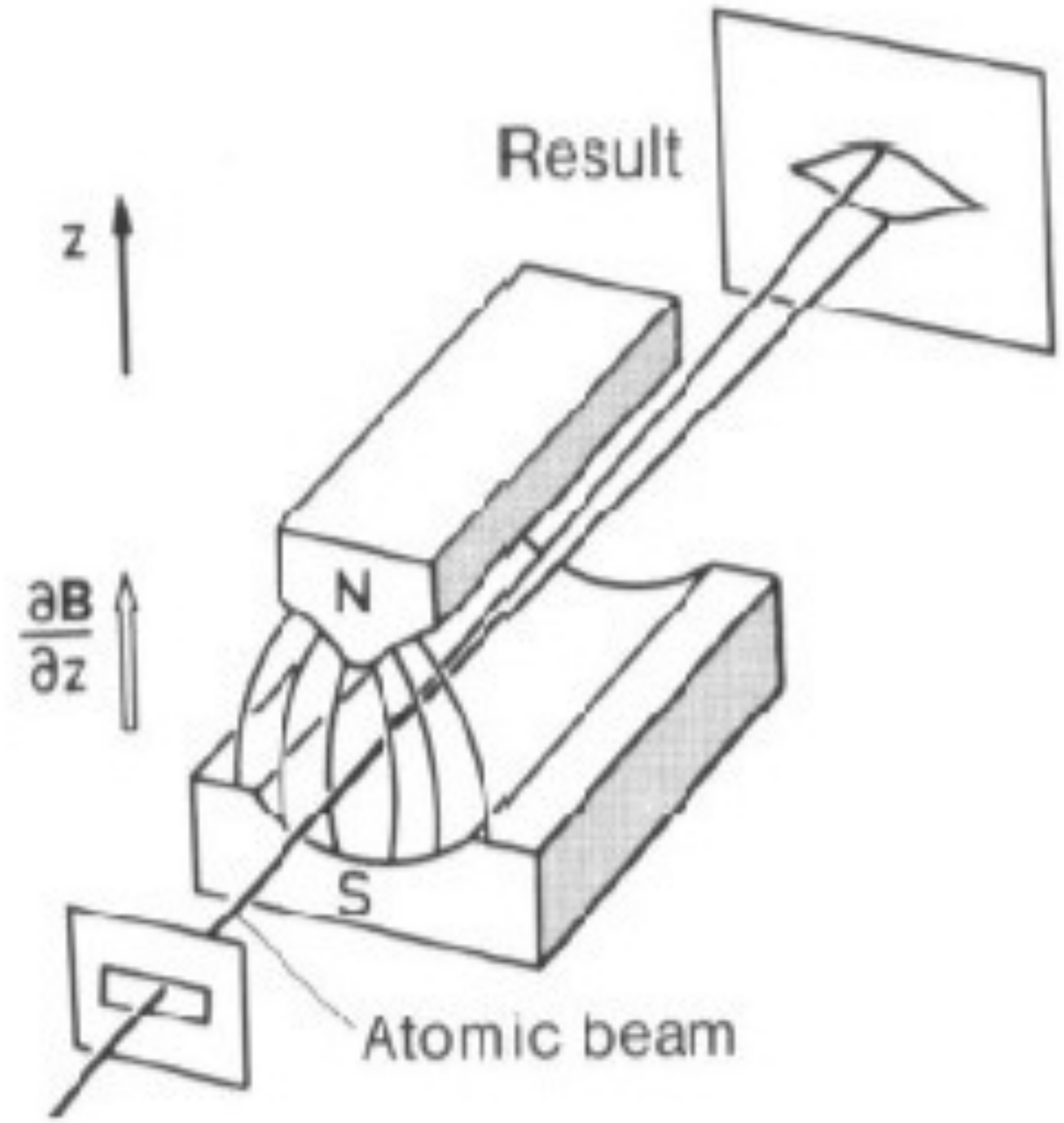






What about situations in which one has only one realization?

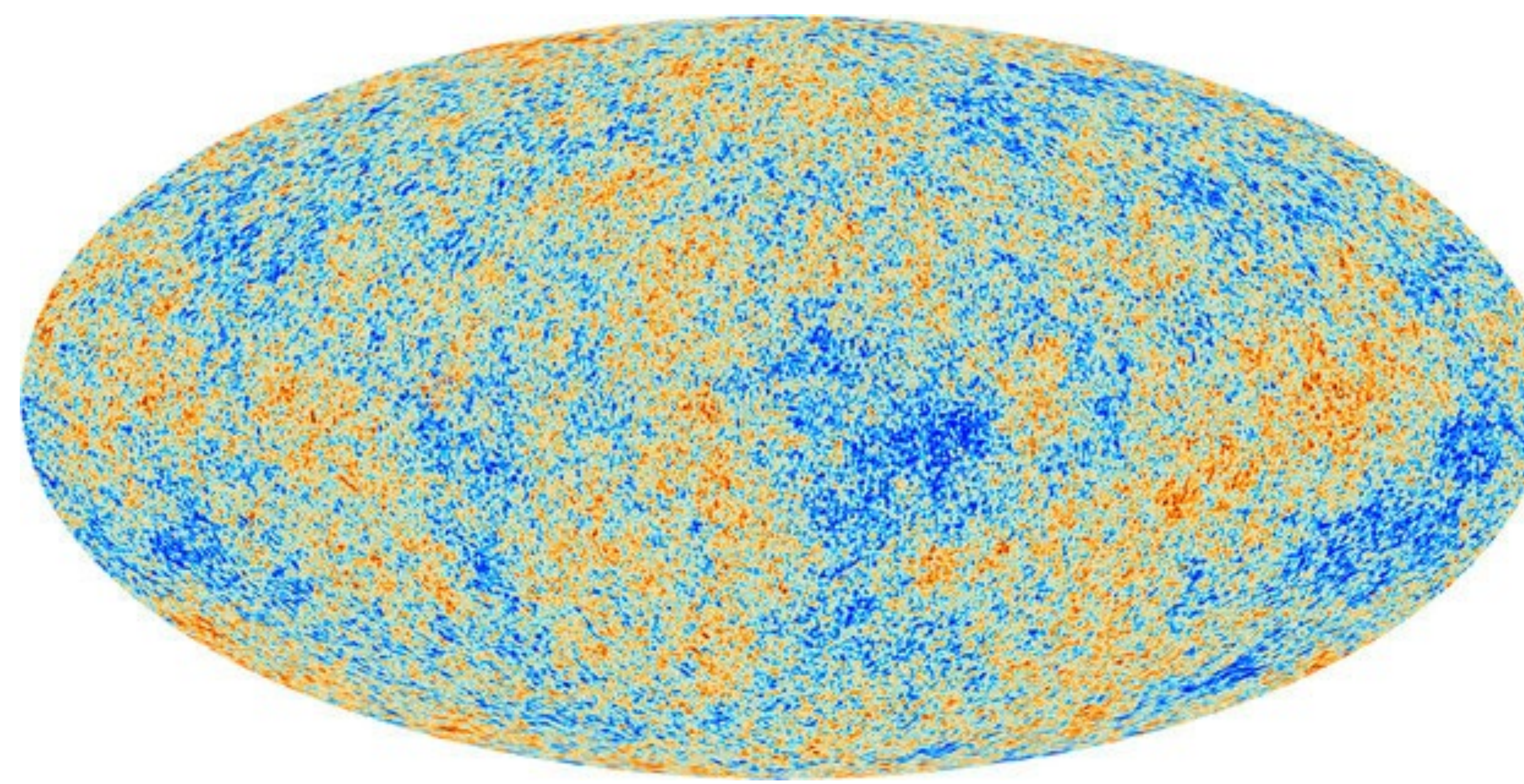
The measurement problem in quantum mechanics



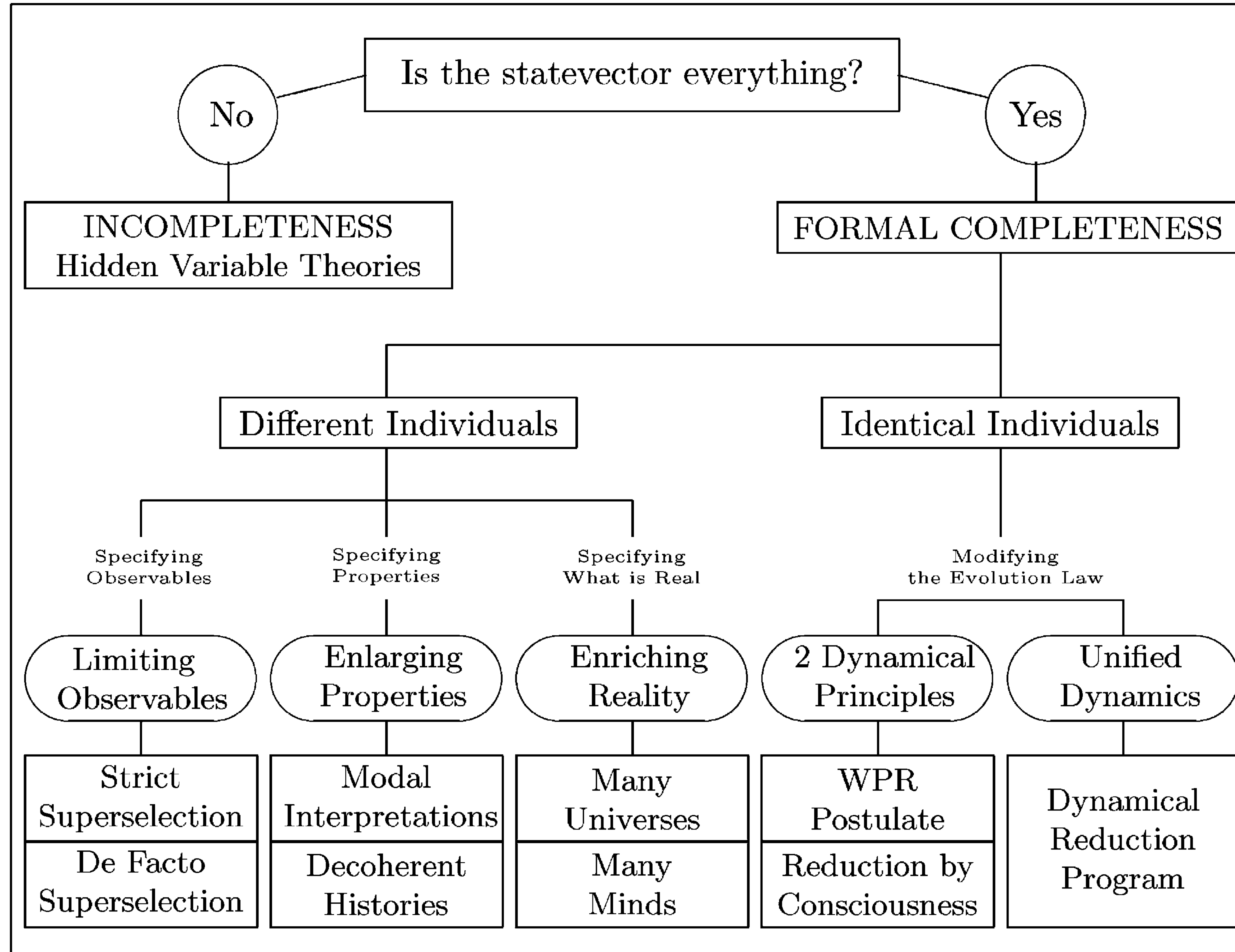
Stern-Gerlach

What about situations in which one has only one realization?

What about the Universe itself?



• Possible extensions and a criterion: the Born rule



A. Bassi & G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

- ▲ ***Superselection rules***
- ▲ ***Modal interpretation***
- ▲ ***Consistent histories***
- ▲ ***Many worlds / many minds***

- ▲ ***Hidden variables***
- ▲ ***Modified Schrödinger dynamics***

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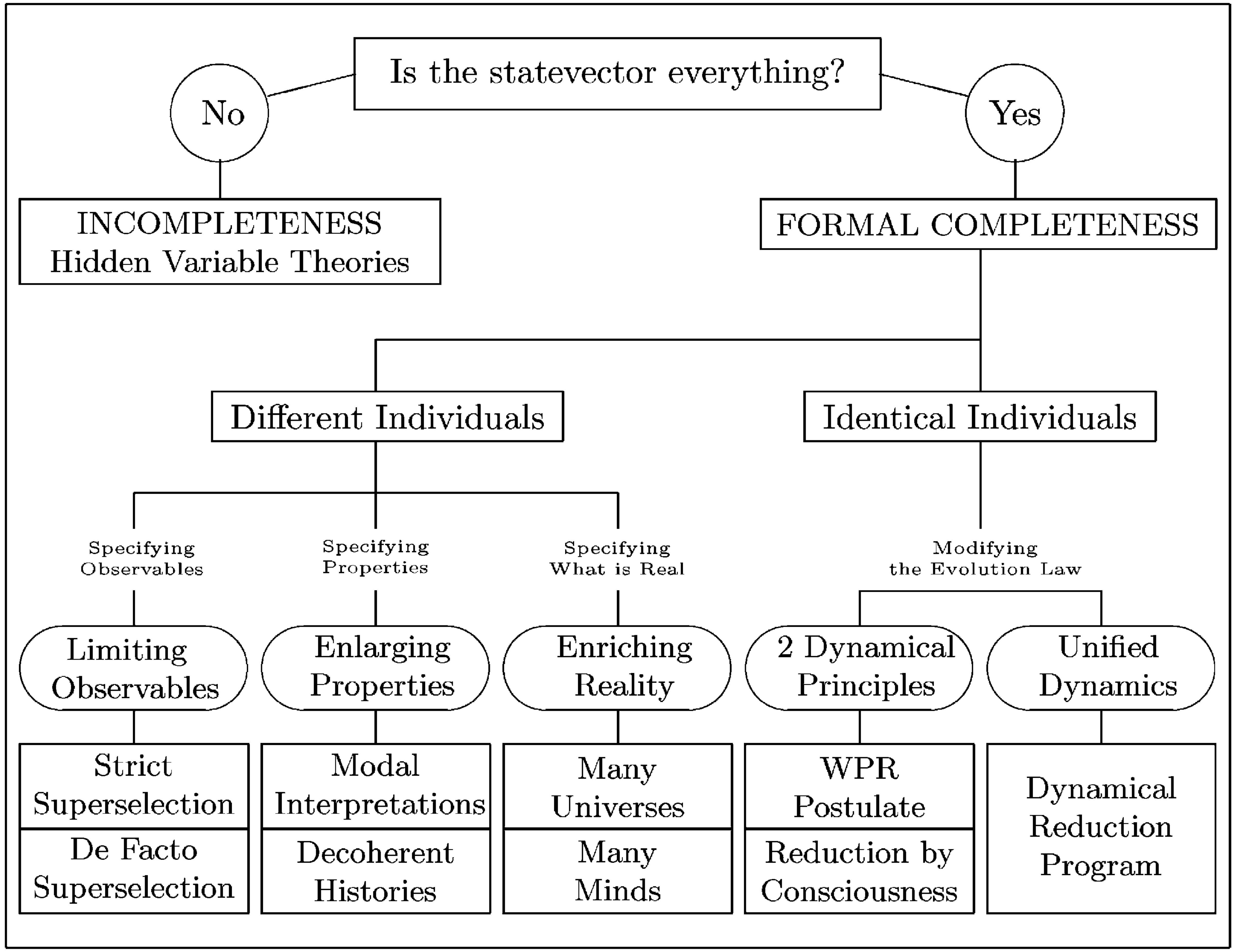

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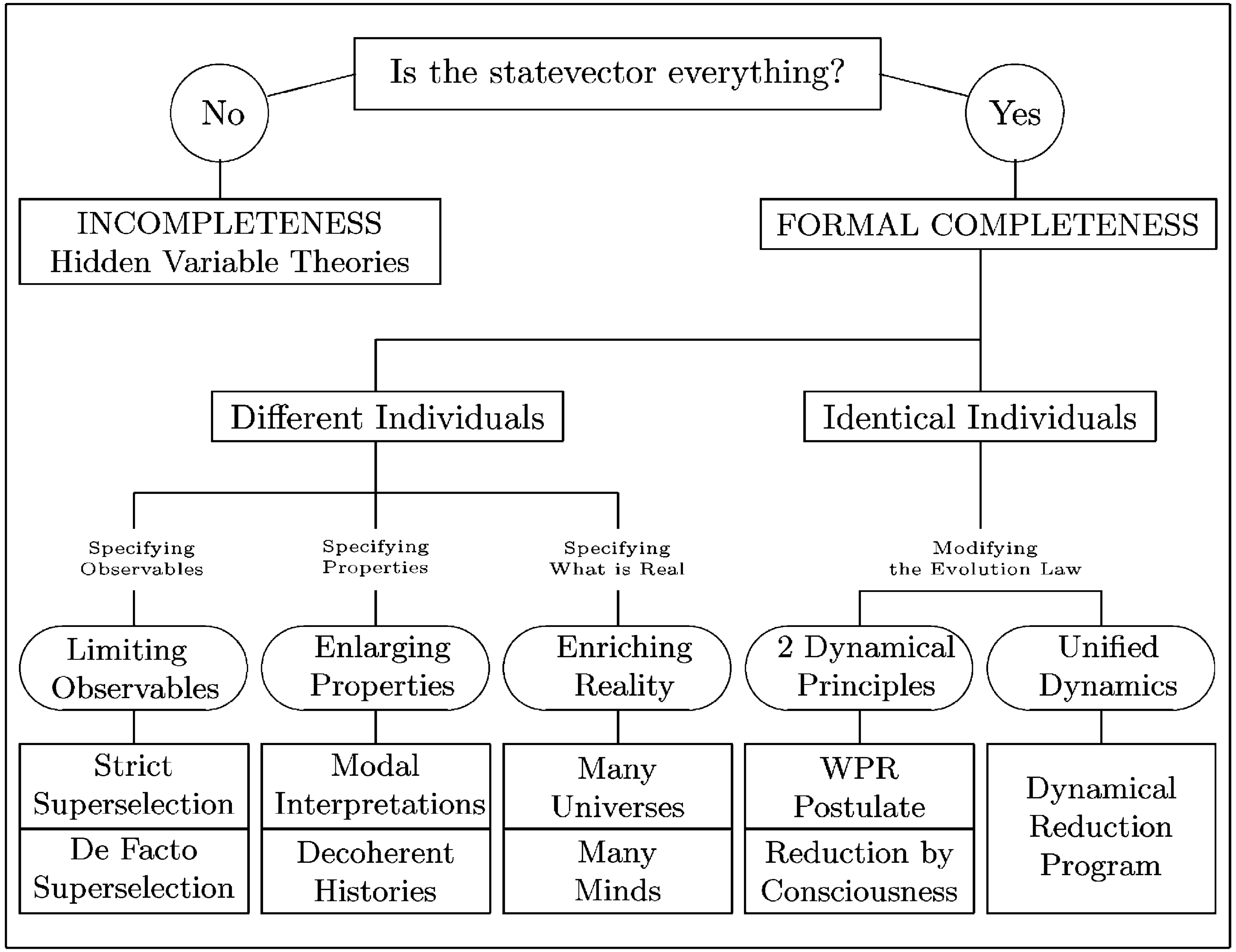

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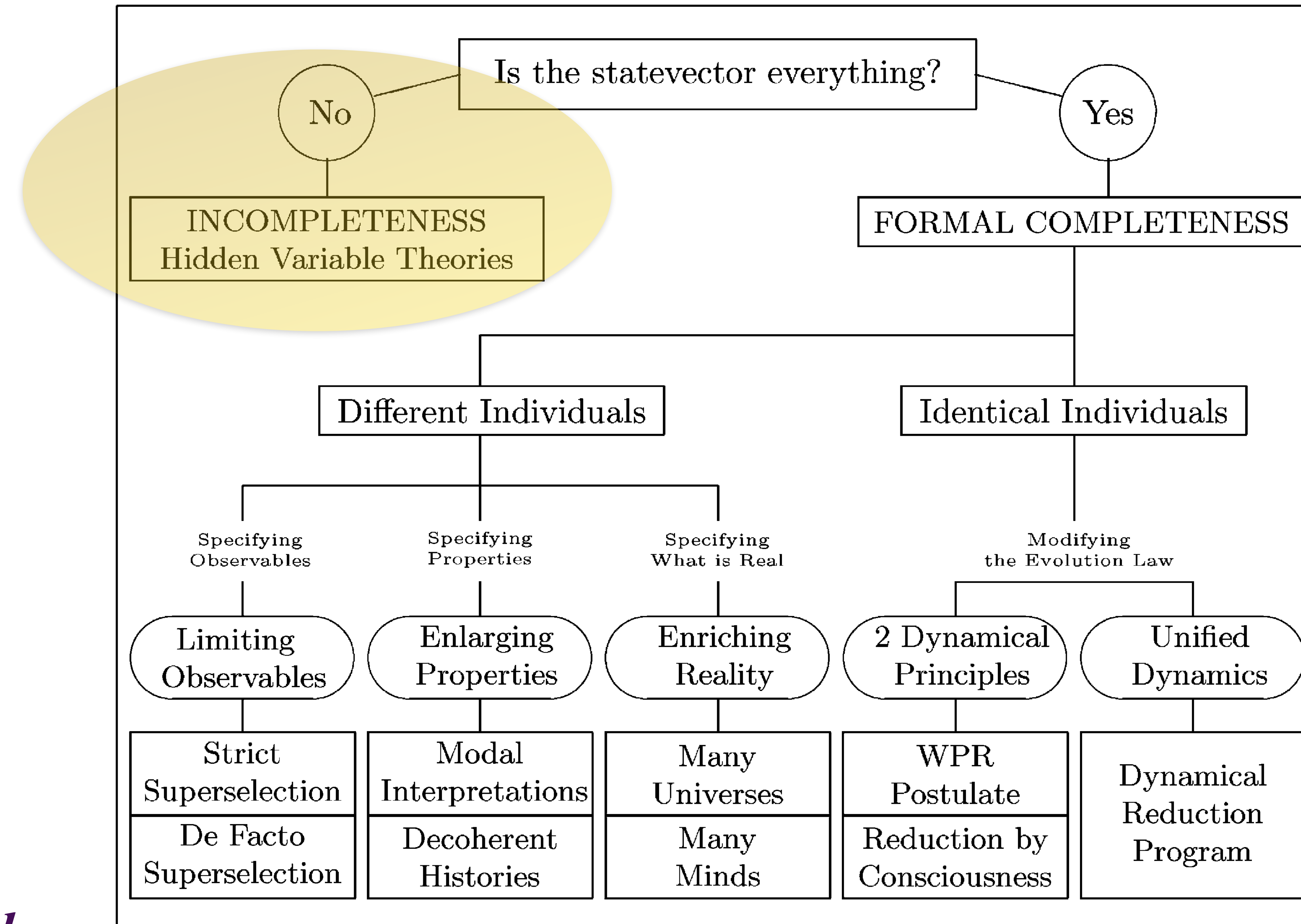

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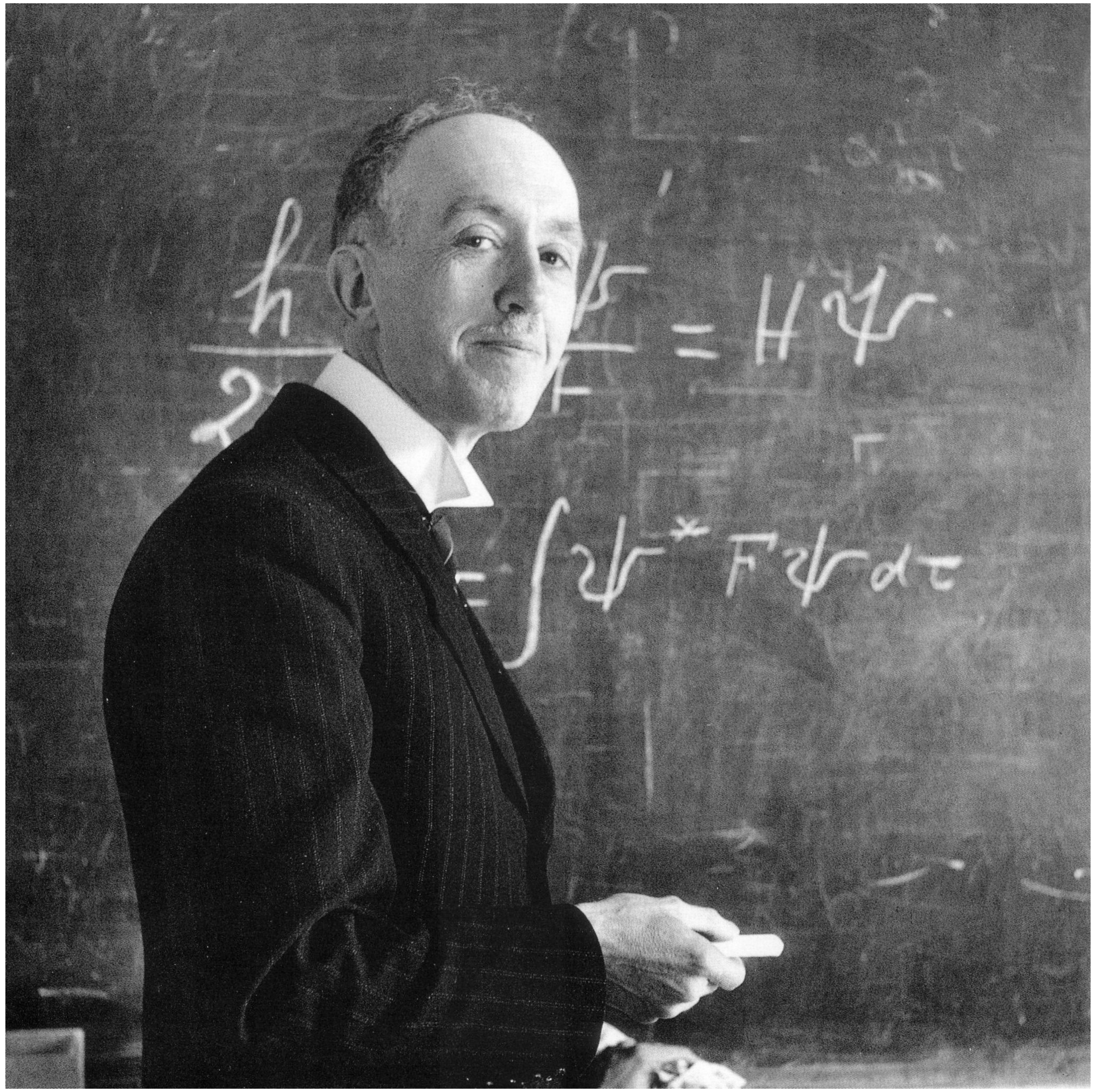
Ontological interpretation (dBB)



Louis de Broglie (1892 - 1987)

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1927 Solvay meeting and von Neuman “5th assumption”...

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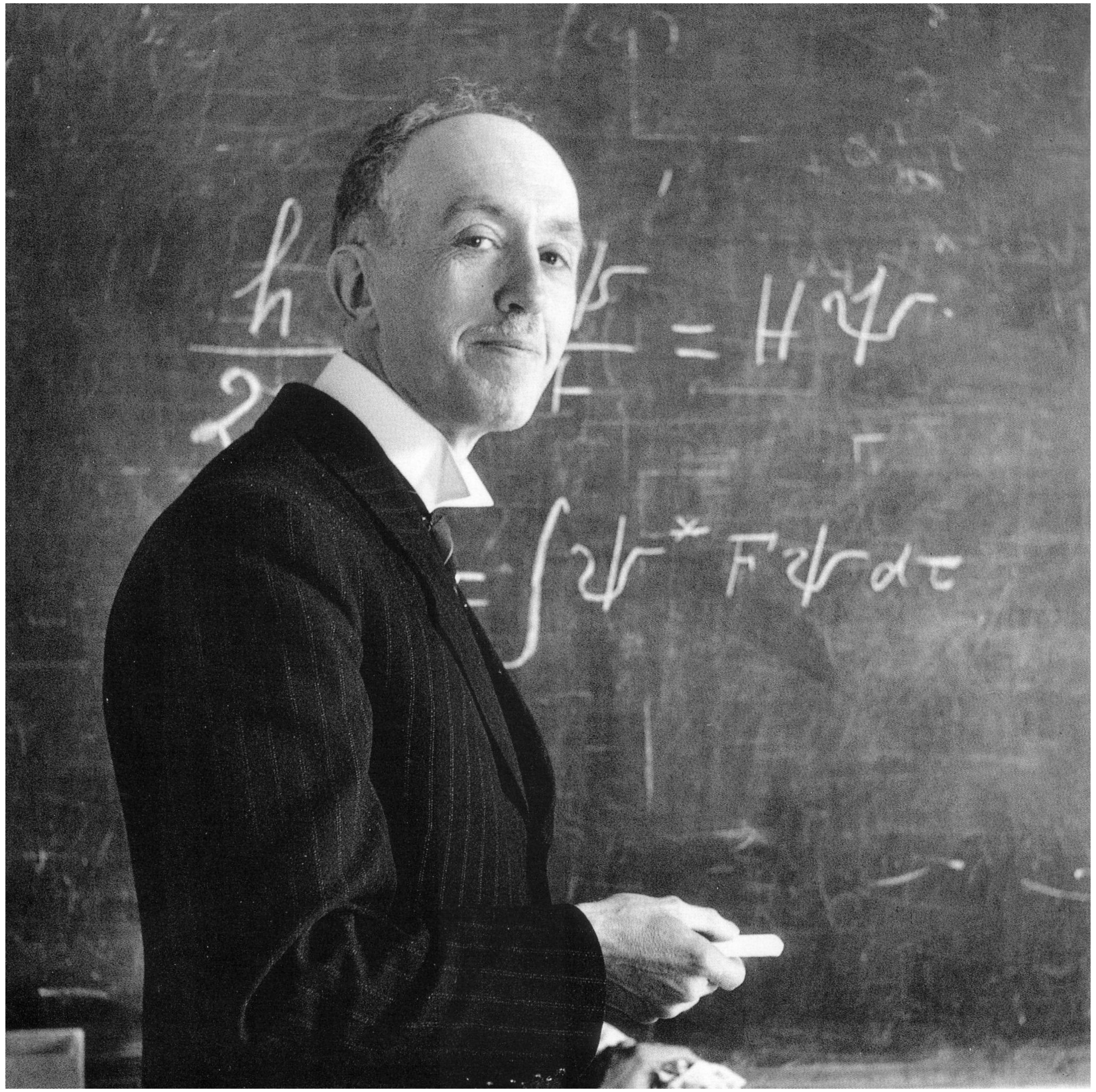

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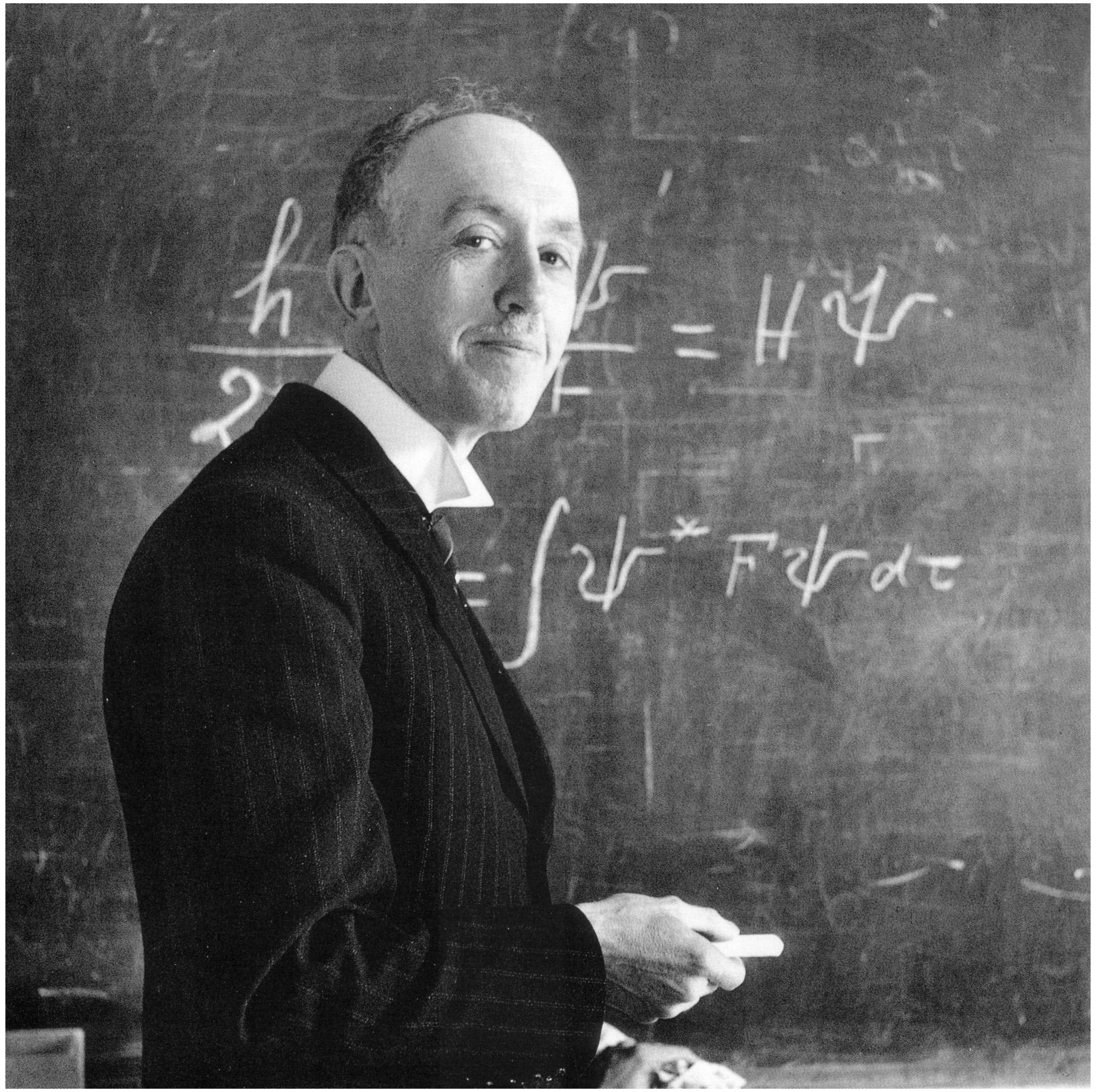

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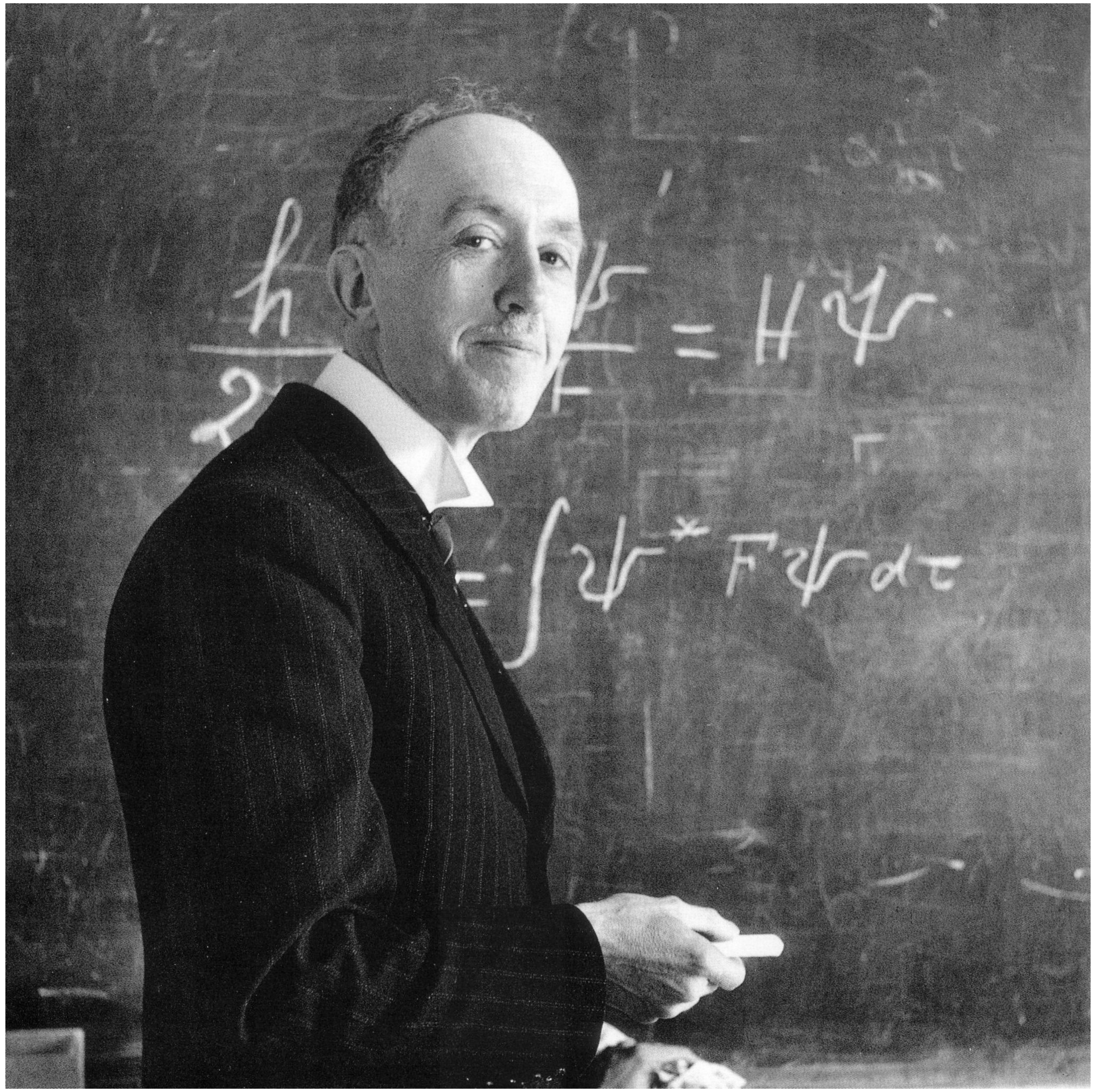
Trajectory formulation **(dBB)**



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Trajectory approach to QM

(1) ordinary QM

Schrödinger equation $$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V(r)\right]\Psi$$

Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\mathrm{e}^{iS(\boldsymbol{r},t)/\hbar}$$

→ from now on, $\hbar = 1$

Modified Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential $$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Trajectory approach to QM

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB)

$\exists \boldsymbol{r}(t)$ trajectory satisfying
(de Broglie pilot wave eq.)

$$m\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \Im\mathrm{m}\frac{\Psi^\star \boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{r},t)|^2} = \boldsymbol{\nabla}S(\boldsymbol{r},t)$$

Ontological *formulation* (BdB)

$$\Psi = A\left(\boldsymbol{r}, t\right) \mathrm{e}^{iS(\boldsymbol{r},t)}$$

$\exists \boldsymbol{r}(t)$ trajectory satisfying
(Bojm modified dynamics)

$$m\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q)$$

$$Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

Trajectory approach to QM

$$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists \boldsymbol{r}(t)$

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Properties:

- ☺ strictly equivalent to Copenhagen QM
  - probability distribution (attractor)
    $\exists t_0; \rho(\boldsymbol{r},t_0)|\Psi(\boldsymbol{r},t_0)|^2$
- ☺ classical limit well defined $Q \longrightarrow 0$ $\left(Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}\right)$
- ☺ state dependent
- ☺ $\exists$ intrinsic reality
  - non local …
- ☺ no need for external classical domain/observer!

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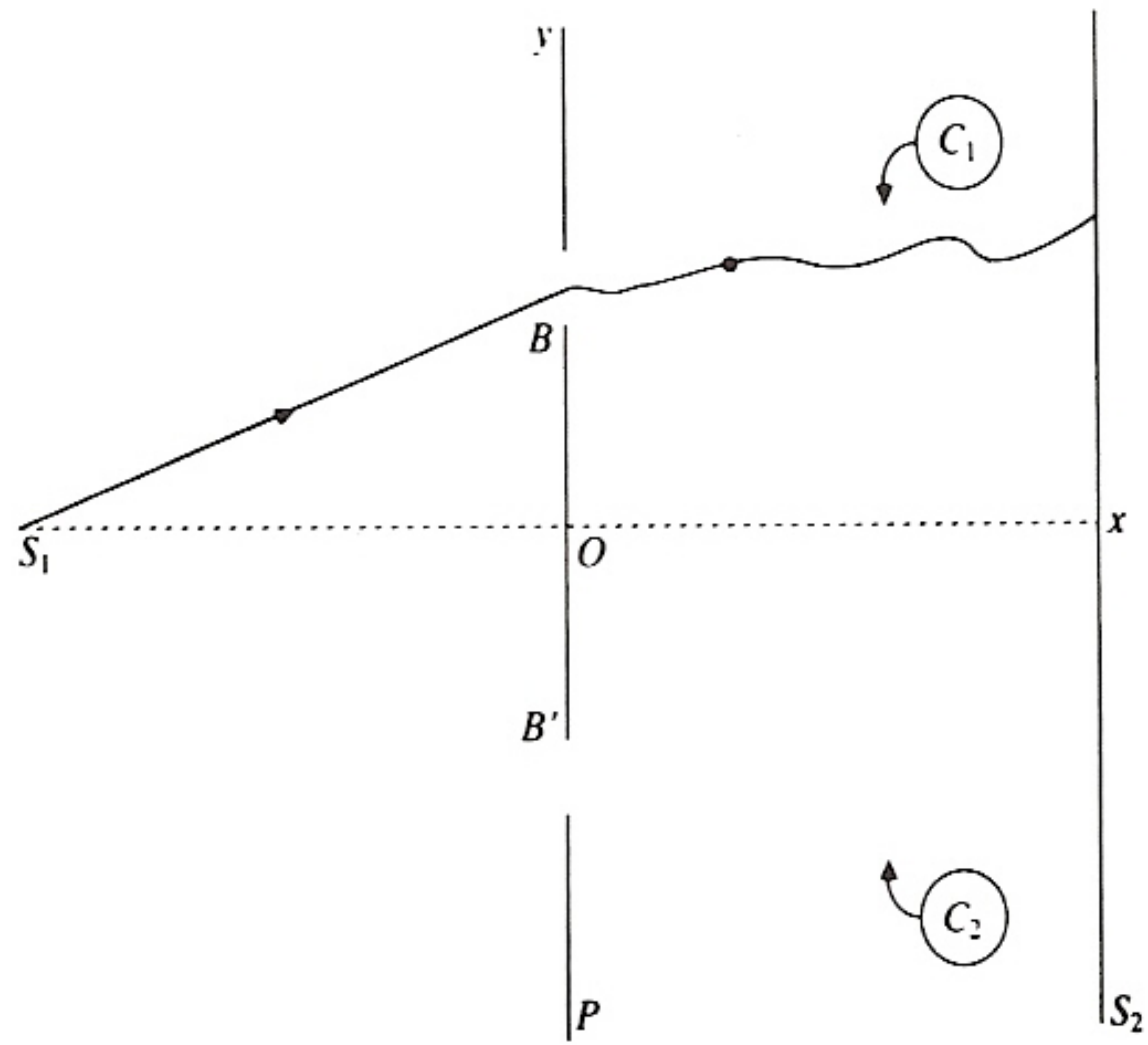
"Bohr brainwashed a whole generation of physicists into thinking that the job was done 50 years ago."

Murray Gell-Mann

Example: the 2-slit experiment

Surrealistic trajectories?



Non straight in vacuum...

$$m \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\nabla \left(V + Q\right)$$

... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

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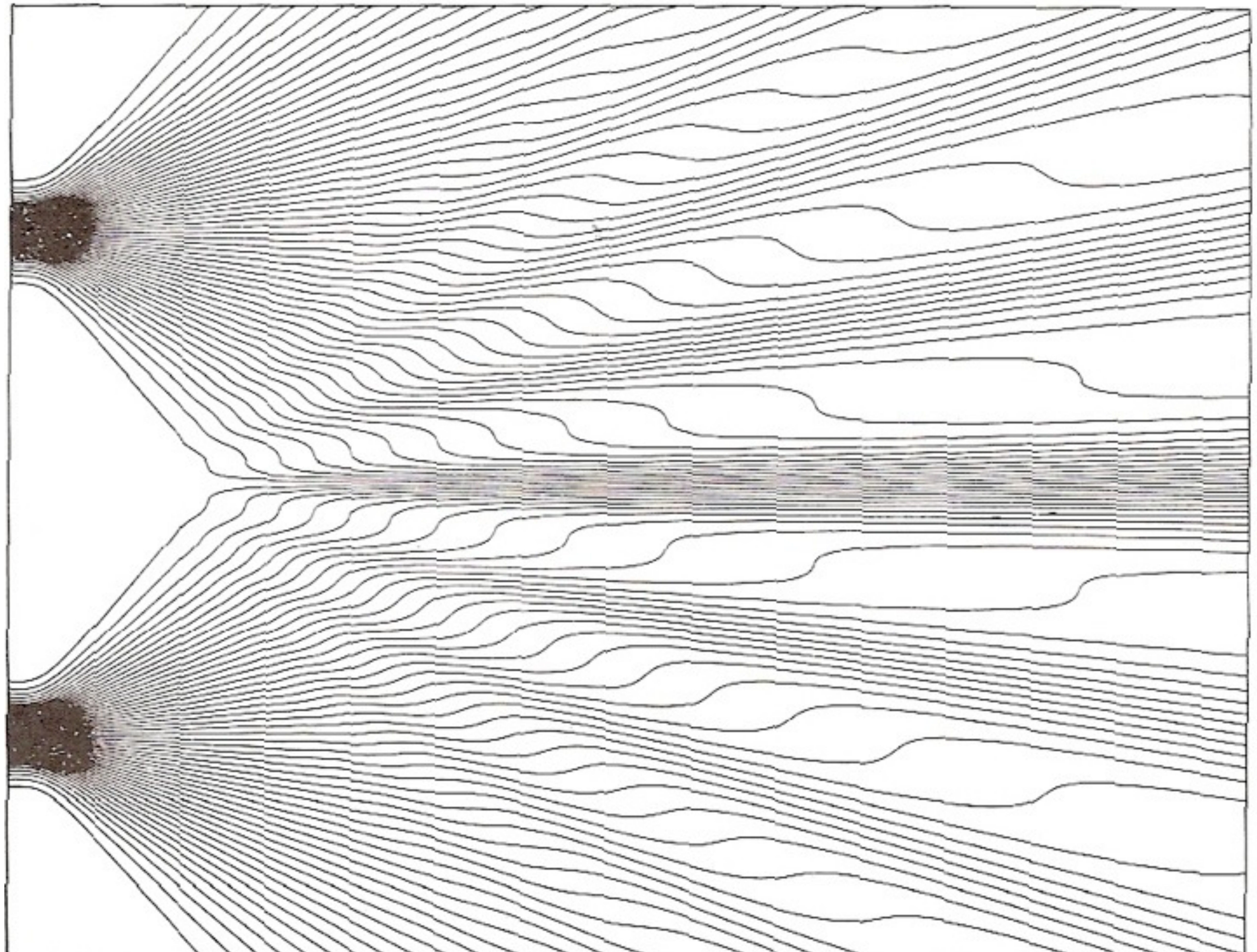

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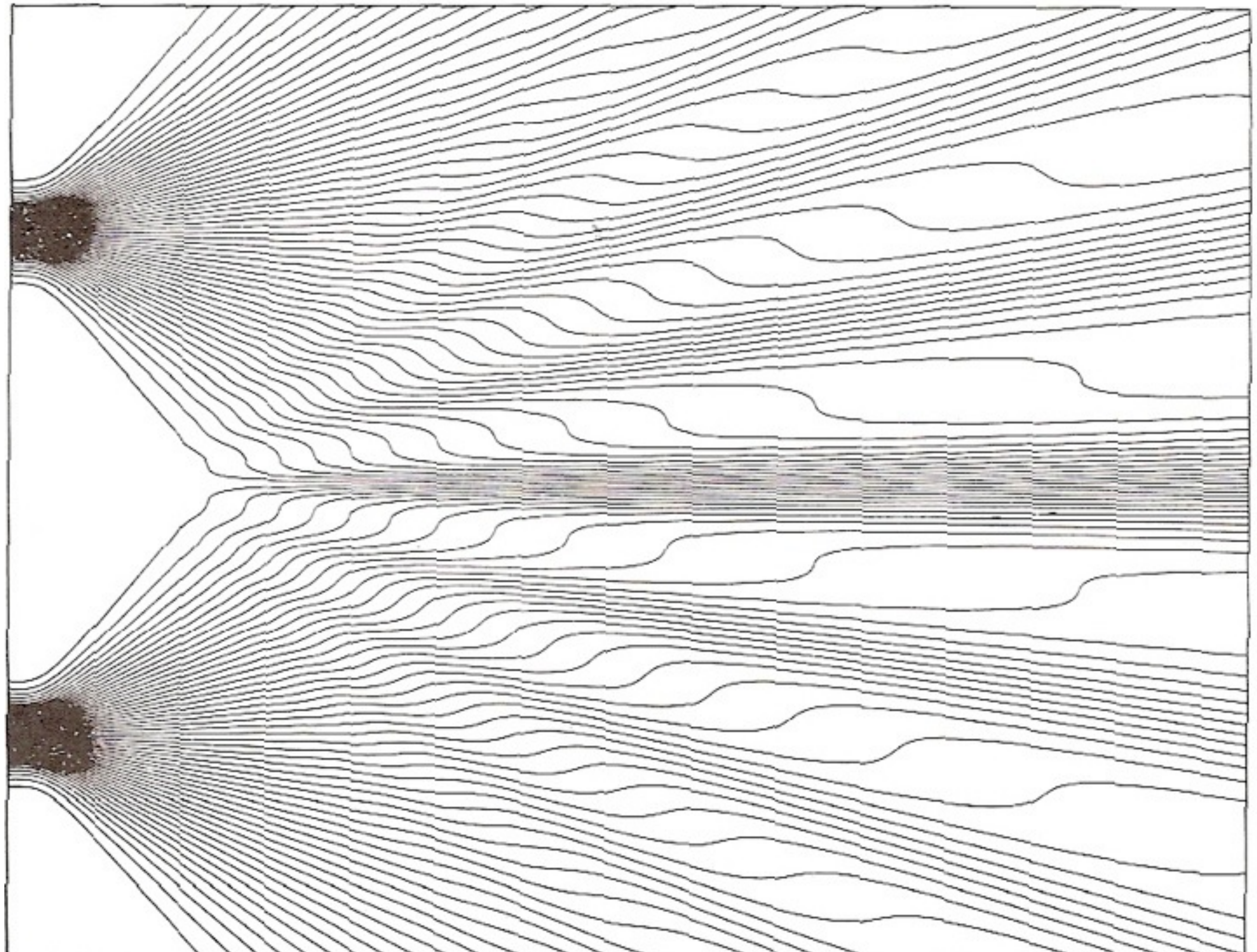

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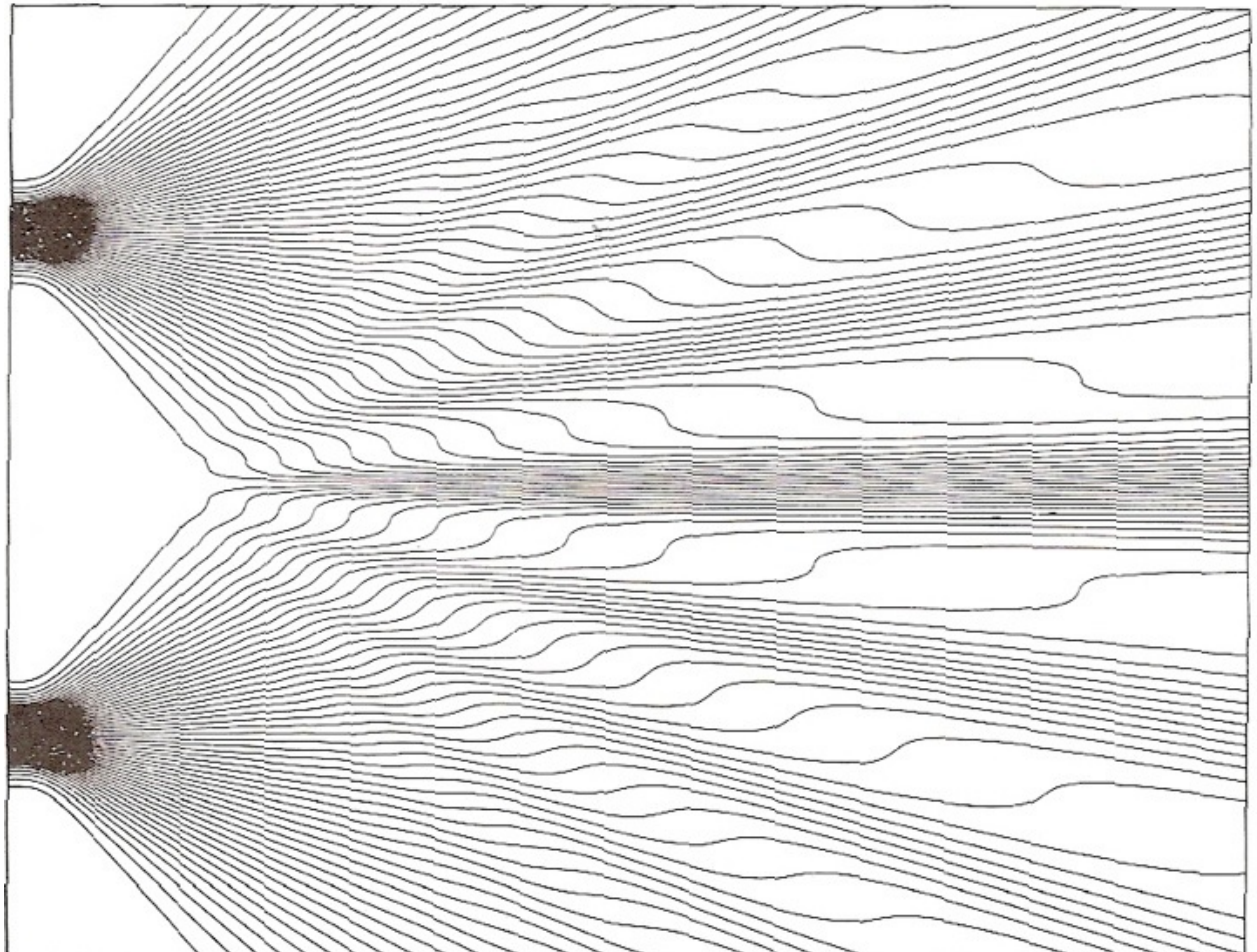

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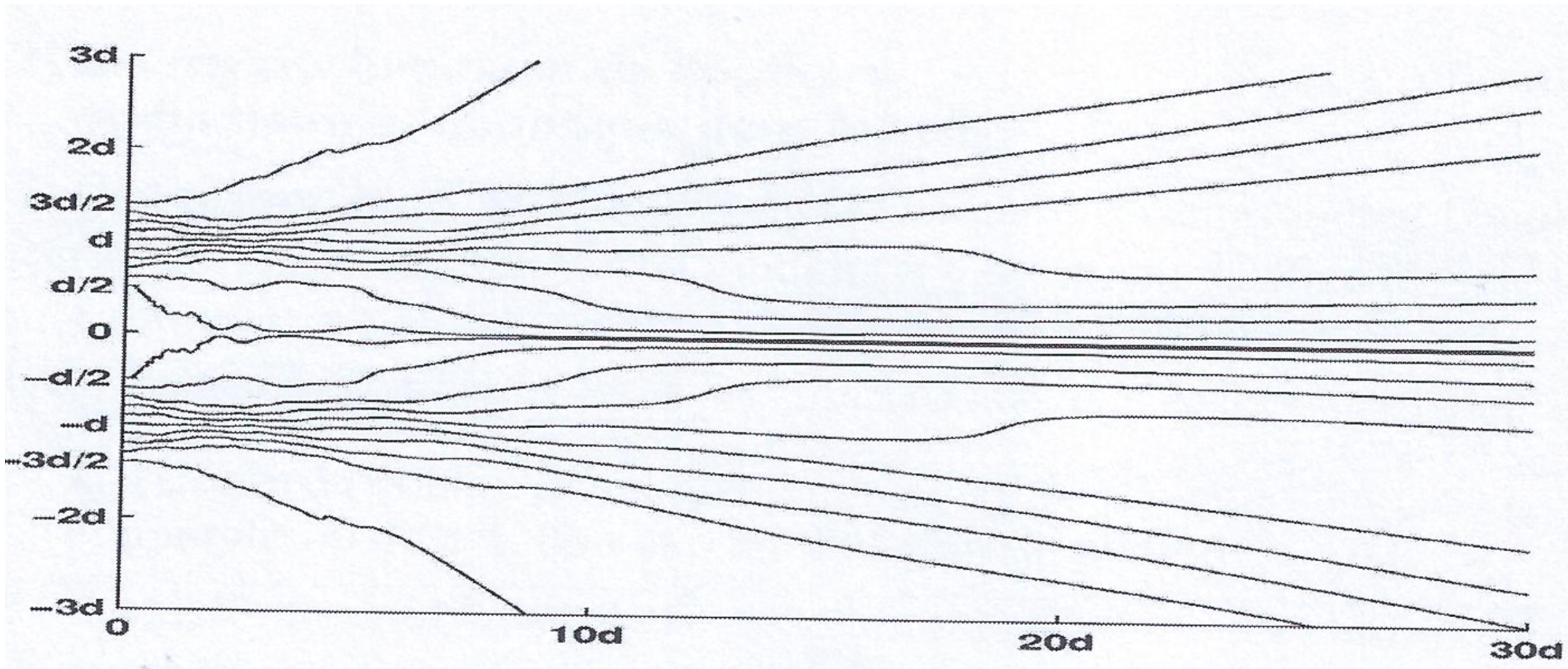

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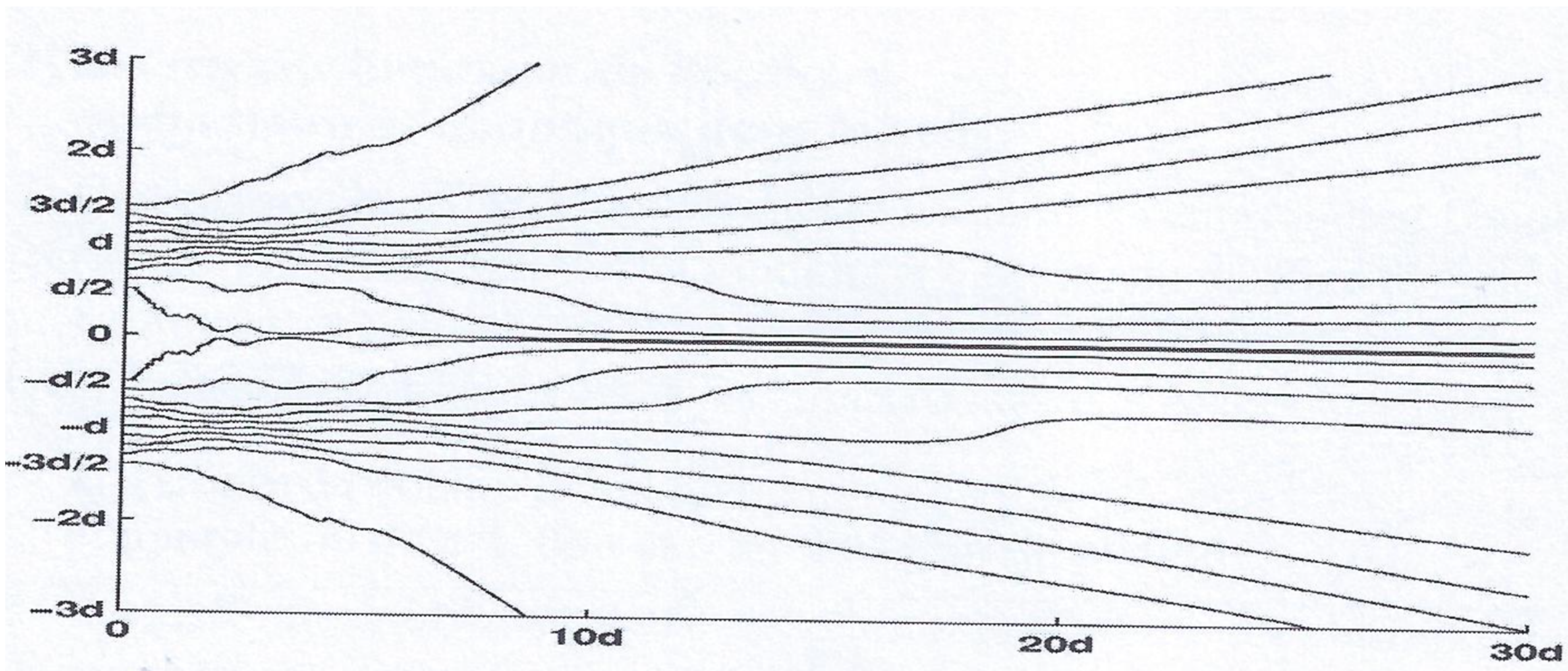

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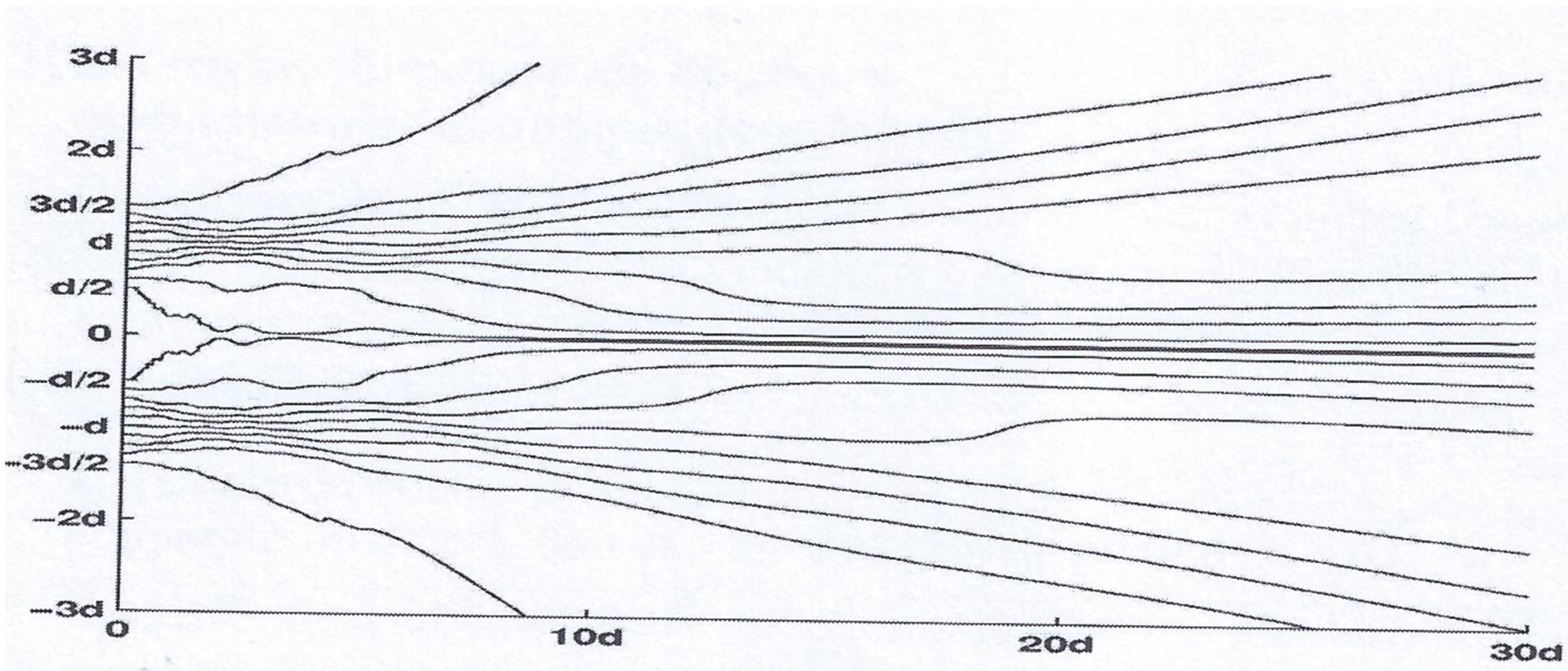

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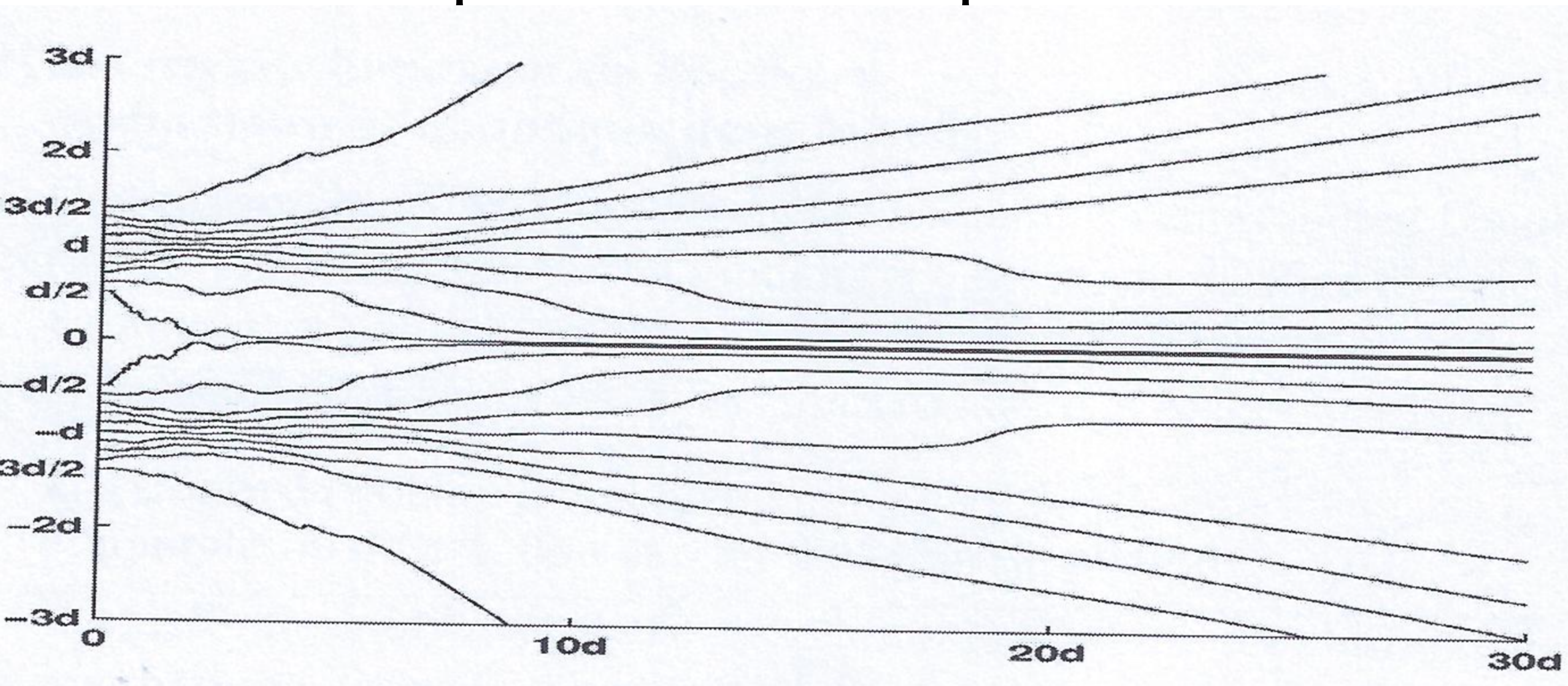
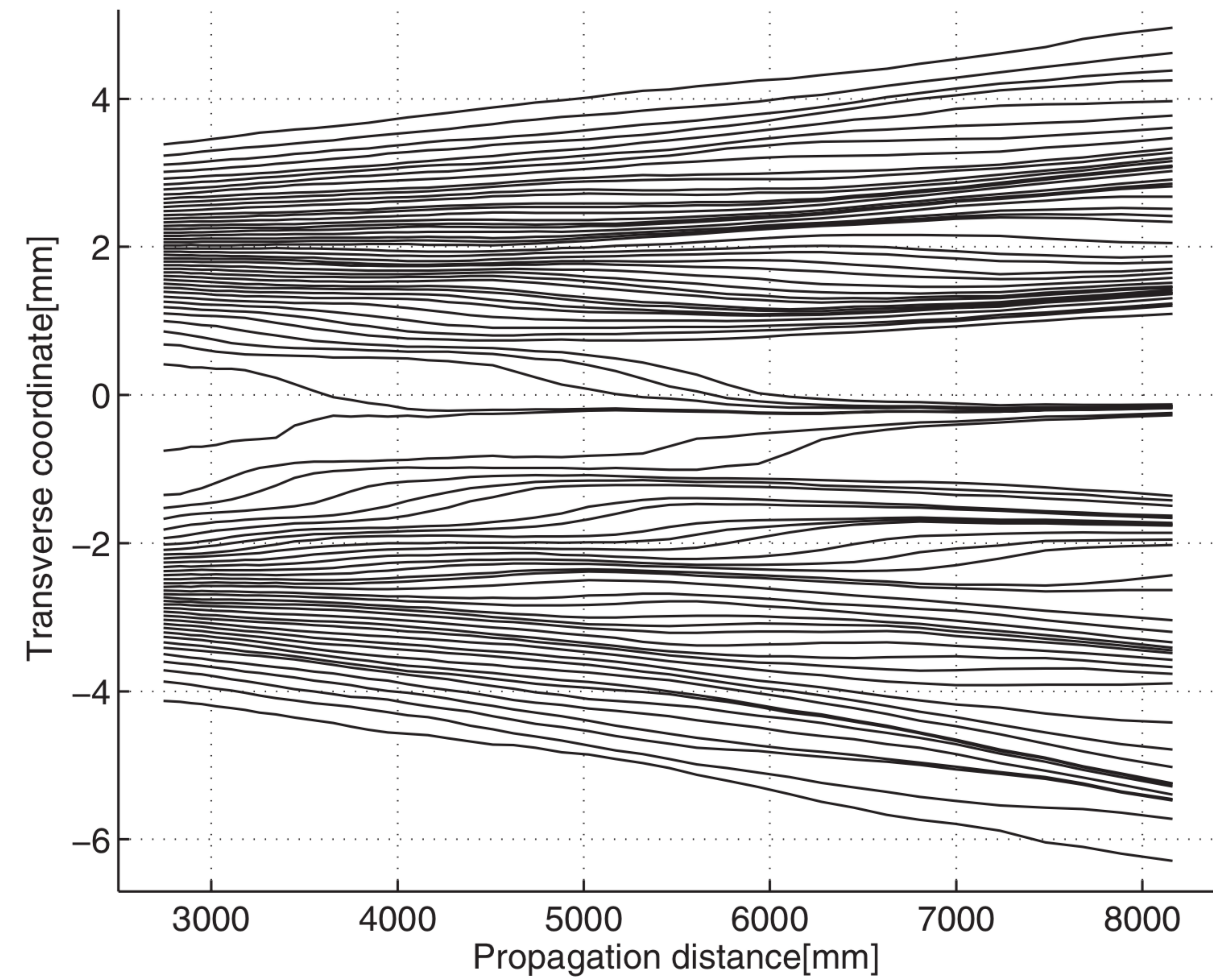


average reconstructed trajectories
(single photons)
weak measurements

S. Kocsis et al., *Science* **332**, 1170 (2011)



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$$i\frac{\partial\Psi}{\partial t} = \frac{1}{4}\frac{\partial^2\Psi}{\partial x^2} \Longleftrightarrow \Psi = \sqrt[4]{\frac{8t_0}{\pi\left(t_0^2+t^2\right)^2}}\exp\left(-\frac{t_0x^2}{t_0^2+t^2}\right)e^{-iS(x,t)}$$

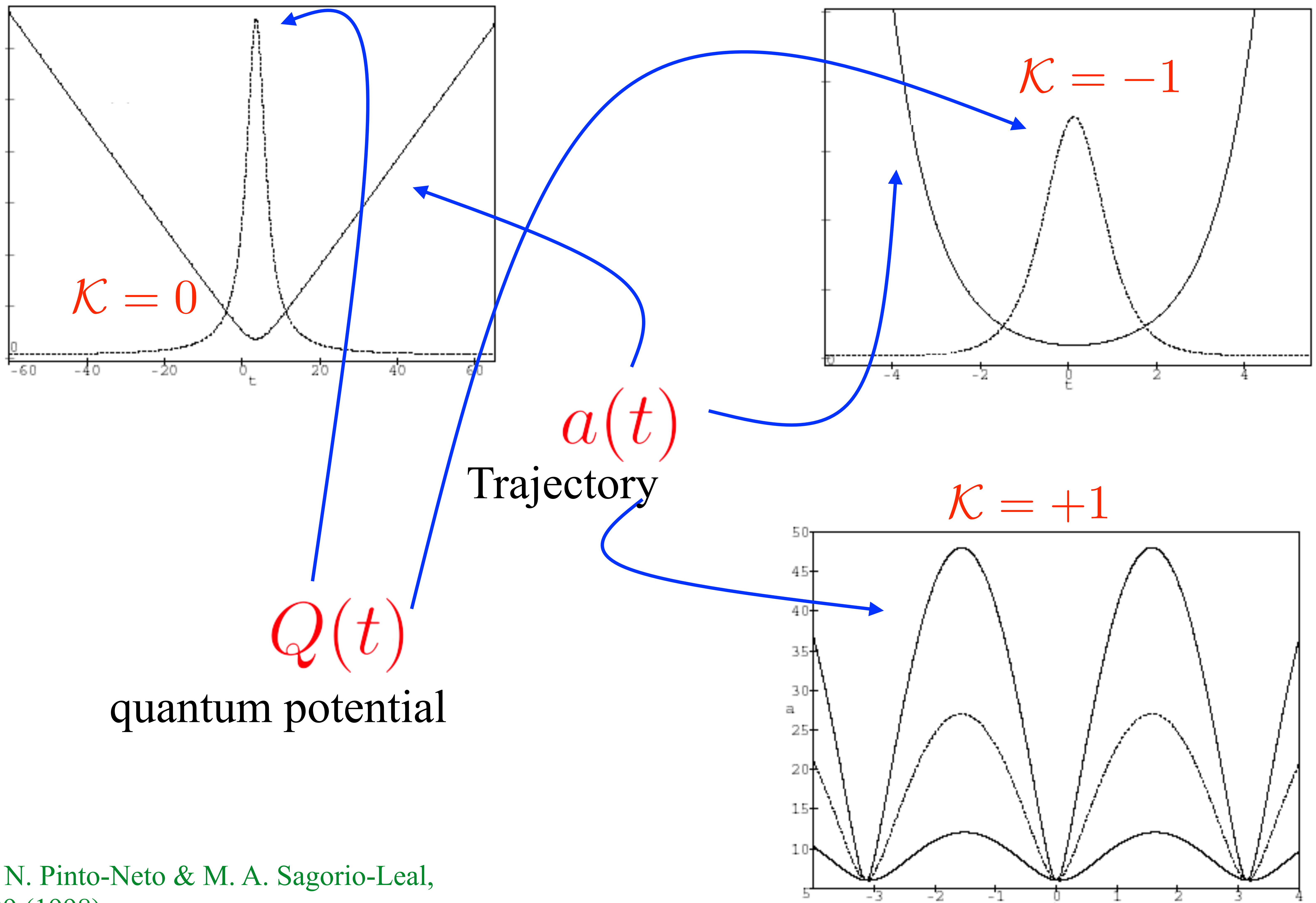
phase $$S(x,t) = \frac{tx^2}{t_0^2+t^2} + \frac{1}{2}\arctan\left(\frac{t_0}{t}\right) - \frac{\pi}{4}$$

Gaussian wave packet

Use dBB trajectory in mini superspace!

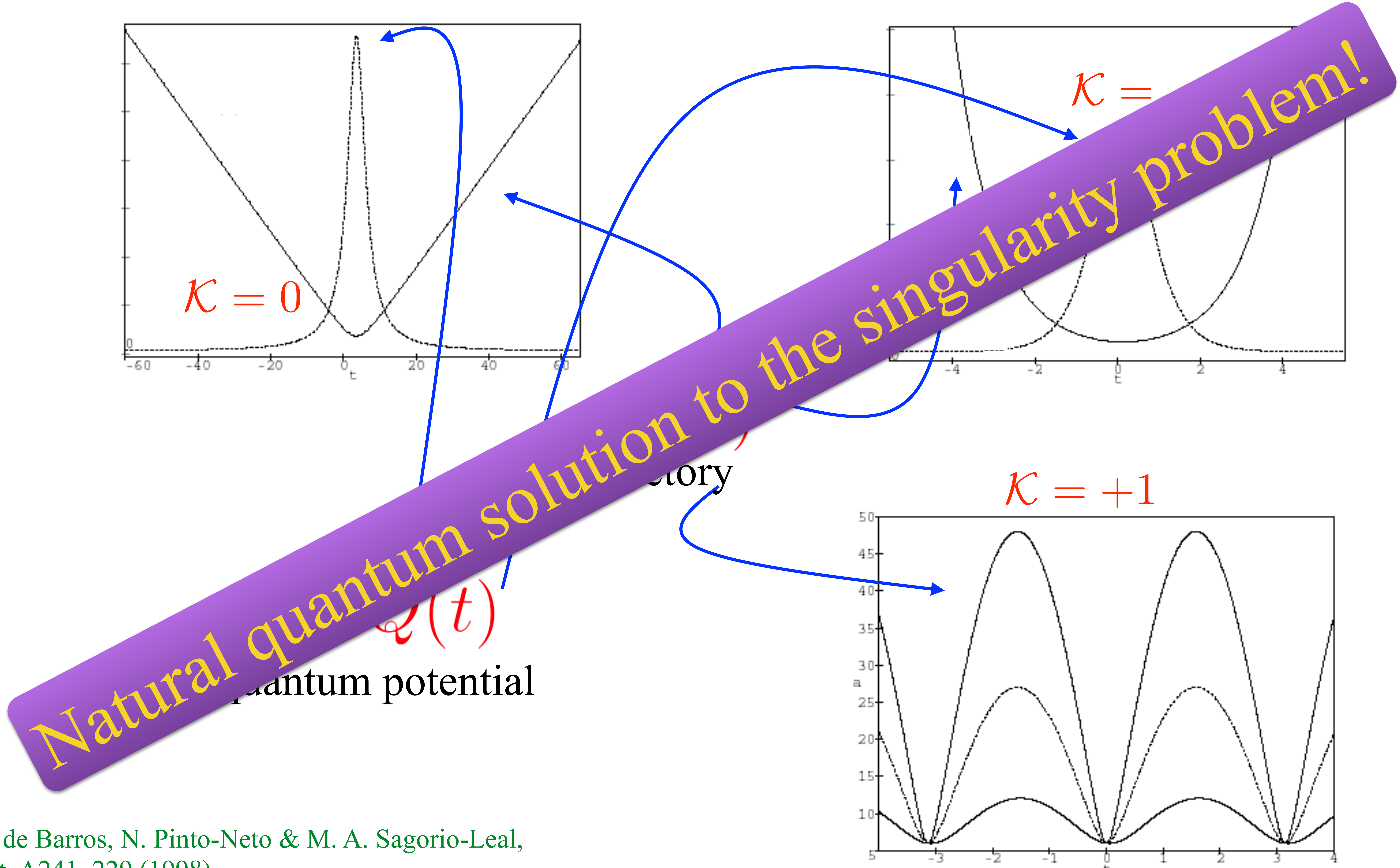
$x \Longleftrightarrow a$ *scale factor*

$$a = a_0\left[1+\left(\frac{t}{t_0}\right)^2\right]^{\frac{1}{3(1-w)}}$$



J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,
Phys. Lett. A241, 229 (1998)

Natural quantum solution to the singularity problem!



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Phys. Lett. A241, 229 (1998)

A simple Bianchi I model $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + \sum_{i=1}^{3} a_i^2(t)\left(\mathrm{d}x^i\right)^2$

+ (radiation) fluid / constant equation of state $w \equiv p/\rho = \frac{1}{3}$

conformal time choice $N \to a$

$t \to \eta$

GR Hamiltonian $H = \frac{\Pi_a^2}{24} - \frac{p_-^2 + p_+^2}{24a^2}$

$a \equiv (a_1 a_2 a_3)^{\frac{1}{3}}$

$\beta_- \equiv \frac{1}{2\sqrt{3}} \ln(a_1/a_2)$

$\beta_+ \equiv \frac{1}{6} \ln(a_1 a_2/a_3^2)$

Canonical commutation relations

$[\hat{a}, \hat{\Pi}_a] = [\hat{\beta}_\pm, \hat{p}_\pm] = i$

Rescaling:

$$\hat{H} = \hat{\Pi}_a^2 - \left(\hat{p}_-^2 + \hat{p}_+^2\right)\hat{a}^{-2}$$

mixed representation for the wave function

$$\begin{cases} \hat{a}\Psi &= a\Psi \\ \hat{p}_\pm\Psi &= p_\pm\Psi \\ \hat{\Pi}_a &= -i\partial/\partial a \\ \hat{\beta}_\pm &= i\partial/\partial p_\pm \end{cases}$$

Hilbert space $\mathbb{H}$

$$\mathbb{H} \sim \left\{ f(a, p_+, p_-) \in \mathbb{C}; \int_0^\infty \mathrm{d}a \int_{-\infty}^\infty \mathrm{d}p_+ \int_{-\infty}^\infty \mathrm{d}p_- \, |f(a, p_+, p_-)|^2 < \infty \right\}$$

eigenvalue equation $\hat{H}\Psi = \ell^2\Psi$ ⟹ $-\frac{\partial^2 \mathcal{U}_\ell^{(k)}}{\partial a^2} - \frac{k^2}{4a^2}\mathcal{U}_\ell^{(k)} = \ell^2 \mathcal{U}_\ell^{(k)}$

Wave function

$$\Psi(a, p_+, p_-) = \int_0^\infty \mathrm{d}\ell \int_{-\infty}^\infty \mathrm{d}\beta_+ \int_{-\infty}^\infty \mathrm{d}\beta_- \tilde{\Psi}(\ell, \beta_+, \beta_-) \mathrm{e}^{i(\beta_+ p_+ + \beta_- p_-)} \mathcal{U}_\ell^{(k)}(a)$$

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$$k^2 \equiv 4(p_+^2 + p_-^2)$$

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Self-adjoint Hamiltonian

$$\int \mathrm{d}a\, \mathrm{d}^2 p\, (H\Psi)^* \,\Psi = \int \mathrm{d}a\, \mathrm{d}^2 p\, \Psi^* \,(H\Psi)$$

automatically satisfied if

$$\int_0^\infty \mathrm{d}a\, \mathcal{U}_\ell^{(k)*}(a)\mathcal{U}_{\ell'}^{(k)}(a) = \delta(\ell-\ell')$$

$$\int_0^\infty \mathrm{d}\ell \int_{-\infty}^\infty \mathrm{d}\beta_+ \int_{-\infty}^\infty \mathrm{d}\beta_- |\tilde{\Psi}\,(\ell,\beta_\pm)\,|^2 \ell^2 < \infty$$

$\nu = \frac{1}{2}\sqrt{1-k^2}$ general solution for the energy eigenmodes

$$\mathcal{U}_\ell^{(k)}(a) = c_+\sqrt{a\ell}J_\nu(a\ell) + c_-\sqrt{a\ell}J_{-\nu}(a\ell) \quad \begin{cases} c_+ = 1 \text{ and } c_- = 0 \\ c_+ = 0 \text{ and } c_- = 1 \end{cases}$$

Linear fluid momentum

$$\hat{P}_{\text{fluid}} = -i\partial_\eta$$

Schrödinger ⇨ Evolution operator

$$i\frac{\partial U}{\partial \eta} = \hat{H}U$$

Initial gaussian wave function

with S. Vitenti

$$\Psi_0(a) = \langle a, p_\pm | \Psi_0 \rangle = \frac{2^{(1-2\alpha)/4}\ a^\alpha}{\sigma^{\alpha+1/2}\sqrt{\Gamma\left(\alpha+\frac{1}{2}\right)}} \exp\left[-\frac{1}{2}a^2\left(\frac{1}{2\sigma^2} - i\mathcal{H}_{\rm ini}\right)\right]$$

Propagator $G(a, p_\pm, a_0, p_\pm^0) \equiv \langle a, p_\pm | U | a_0, p_\pm^0 \rangle$

$$= \delta^{(2)}(p_\pm - p'_\pm) \int_0^\infty \mathrm{d}\ell\, \mathrm{e}^{-i\ell^2\Delta\eta} \mathcal{U}_\ell^{(k)}(a) \mathcal{U}_\ell^{(k)*}(a')$$

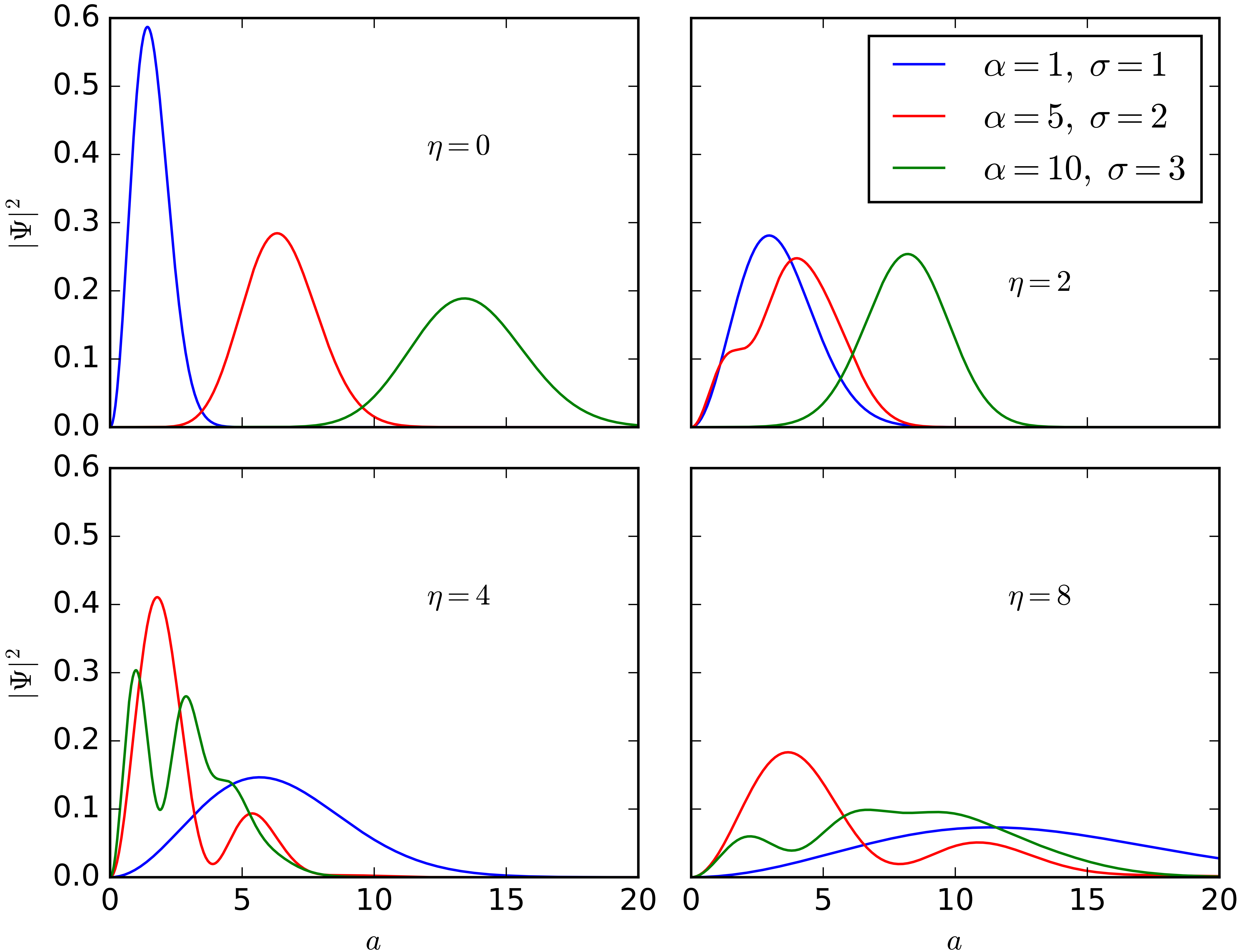
+ regularisation $\widetilde{\Delta\eta} = \Delta\eta(1+i\epsilon)$

$$G(a, a_0; \eta) = -\frac{i\sqrt{aa_0}}{2\widetilde{\Delta\eta}} \mathrm{e}^{\frac{i}{4}(a^2+a_0^2)/\widetilde{\Delta\eta} - i\alpha\pi/2} J_\nu\left(\frac{aa_0}{2\widetilde{\Delta\eta}}\right)$$

dBB trajectory $\dfrac{\mathrm{d}a}{\mathrm{d}\eta} = \dfrac{\partial S}{\partial a} = \dfrac{i}{2|\Psi|^2}\left(\Psi\dfrac{\partial\Psi^*}{\partial a} - \dfrac{\partial\Psi}{\partial a}\Psi^*\right)$

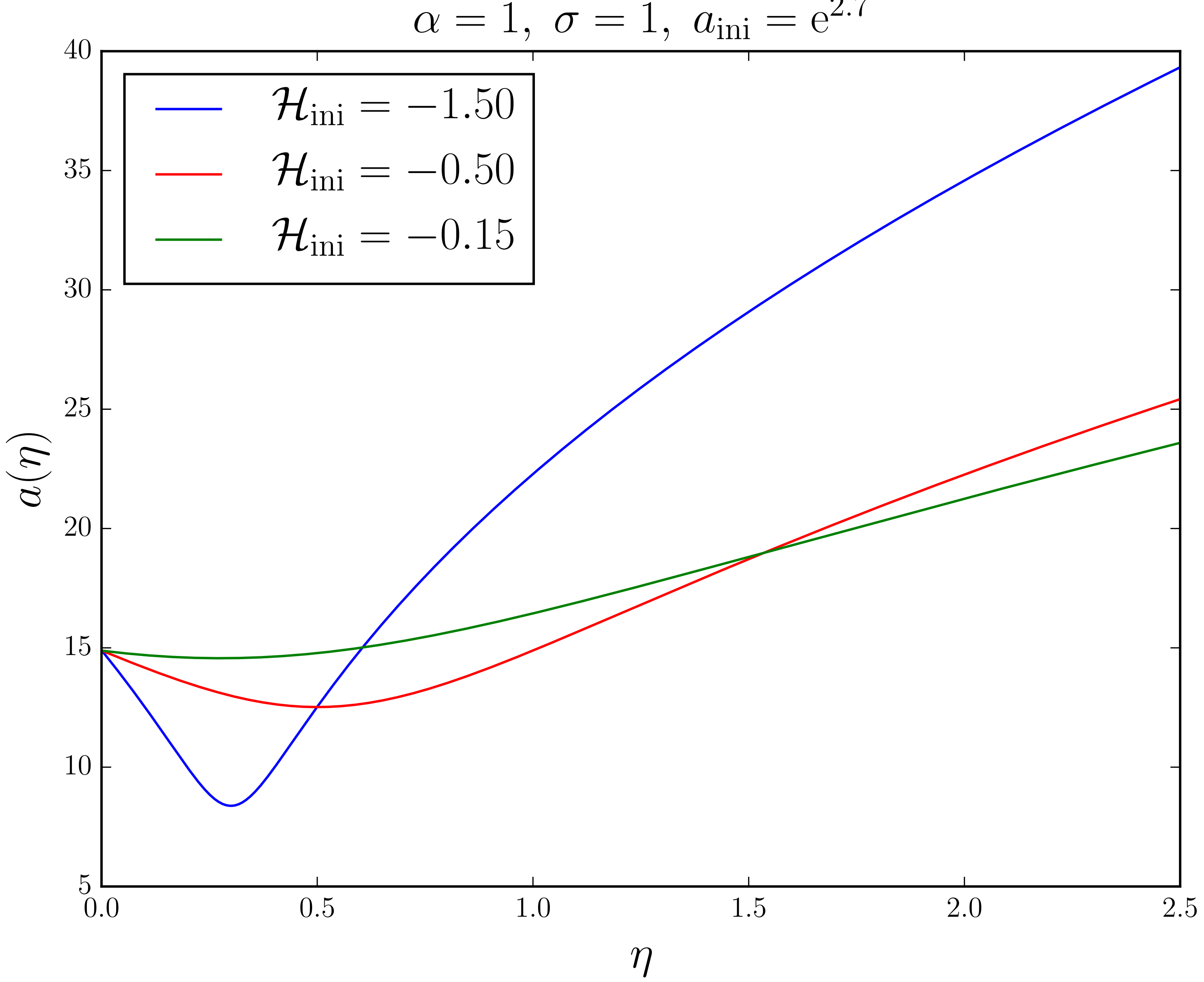
$\nu = \frac{1}{2}$

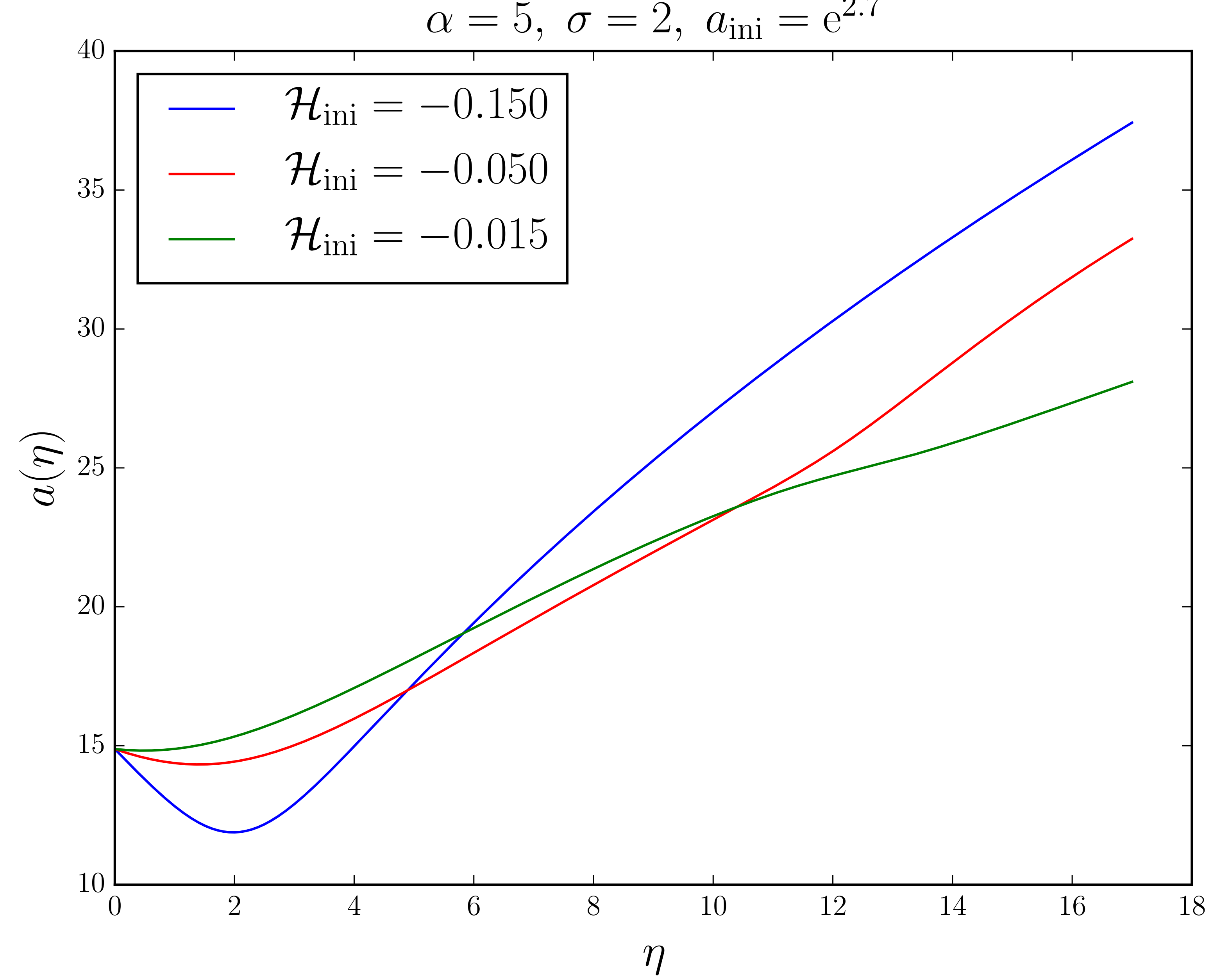
(FLRW)

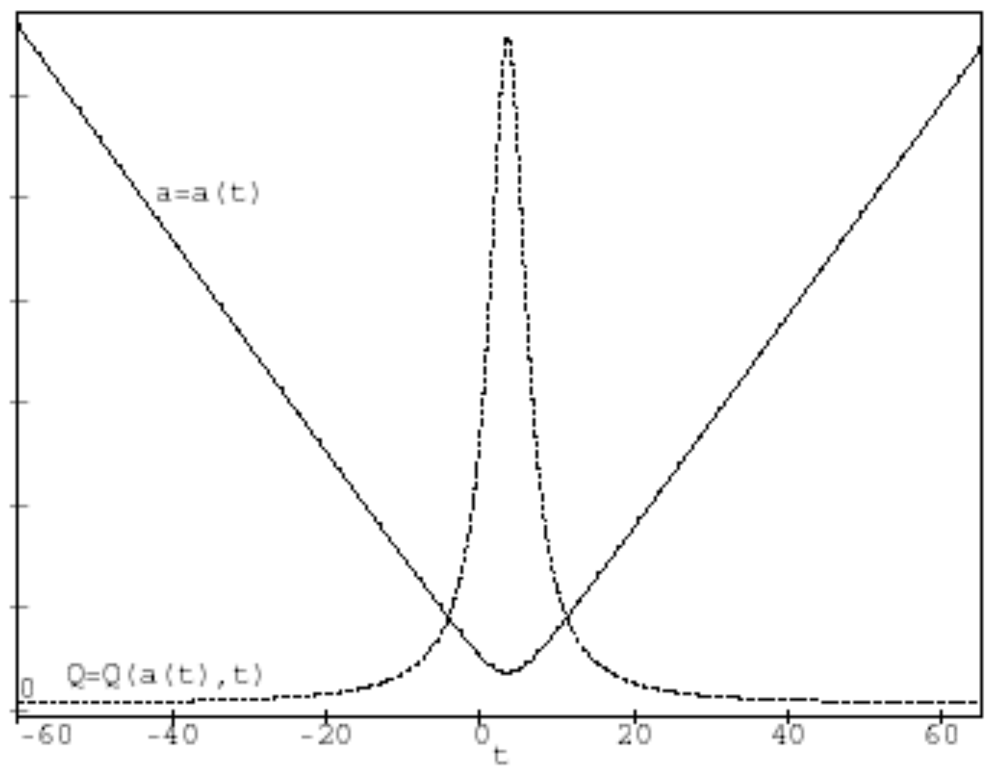


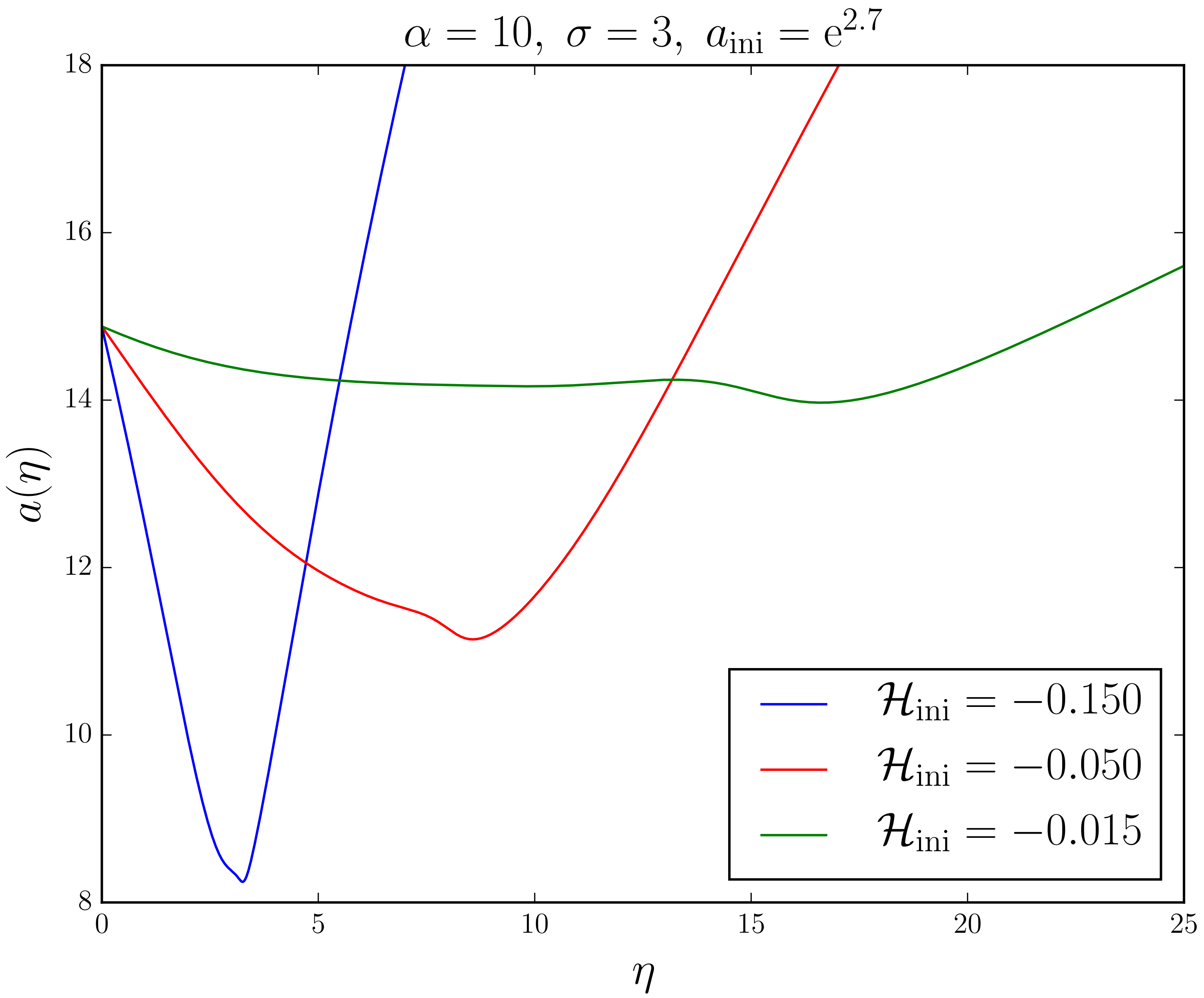
$$\nu = \frac{1}{2}$$

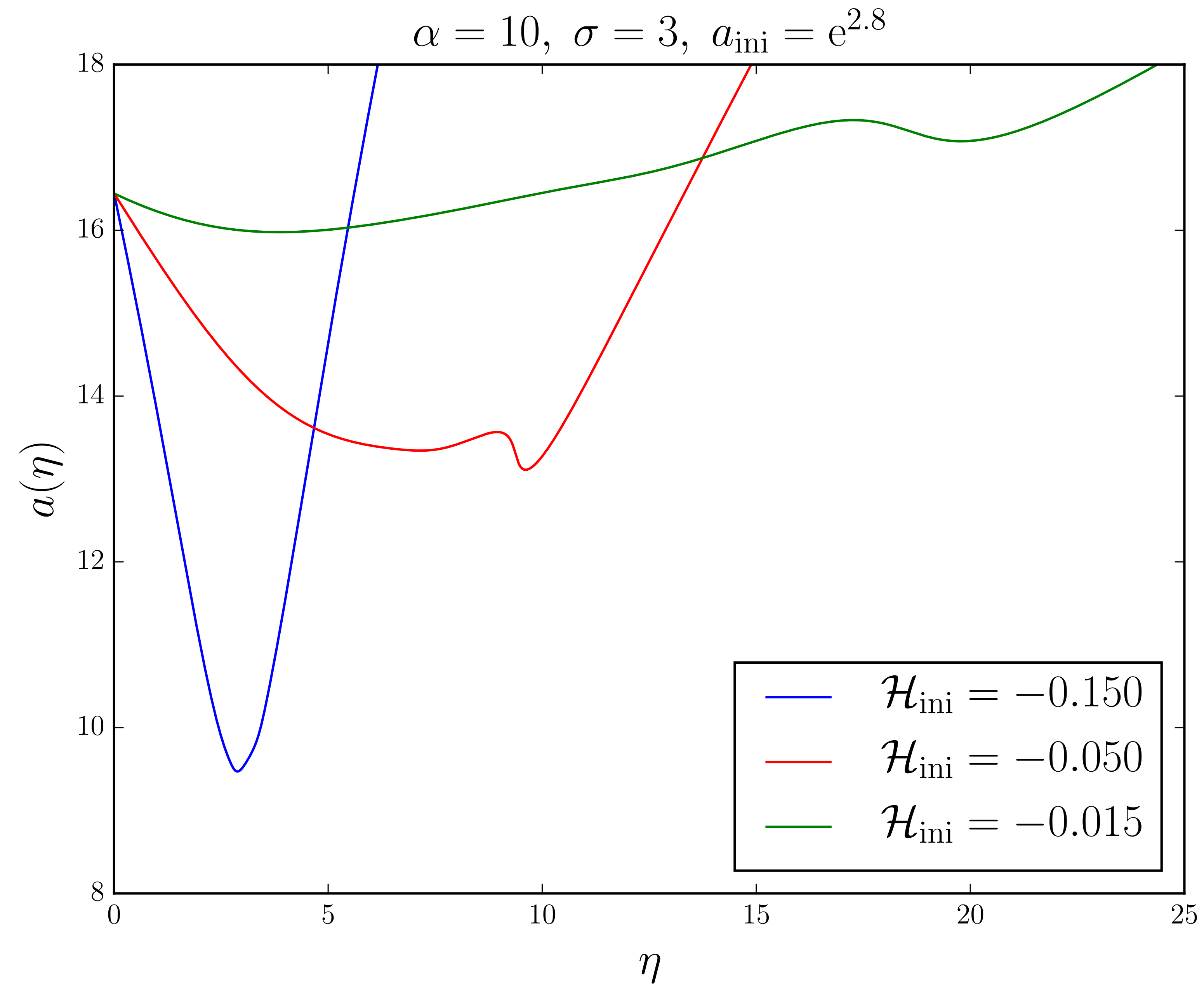
(FLRW)

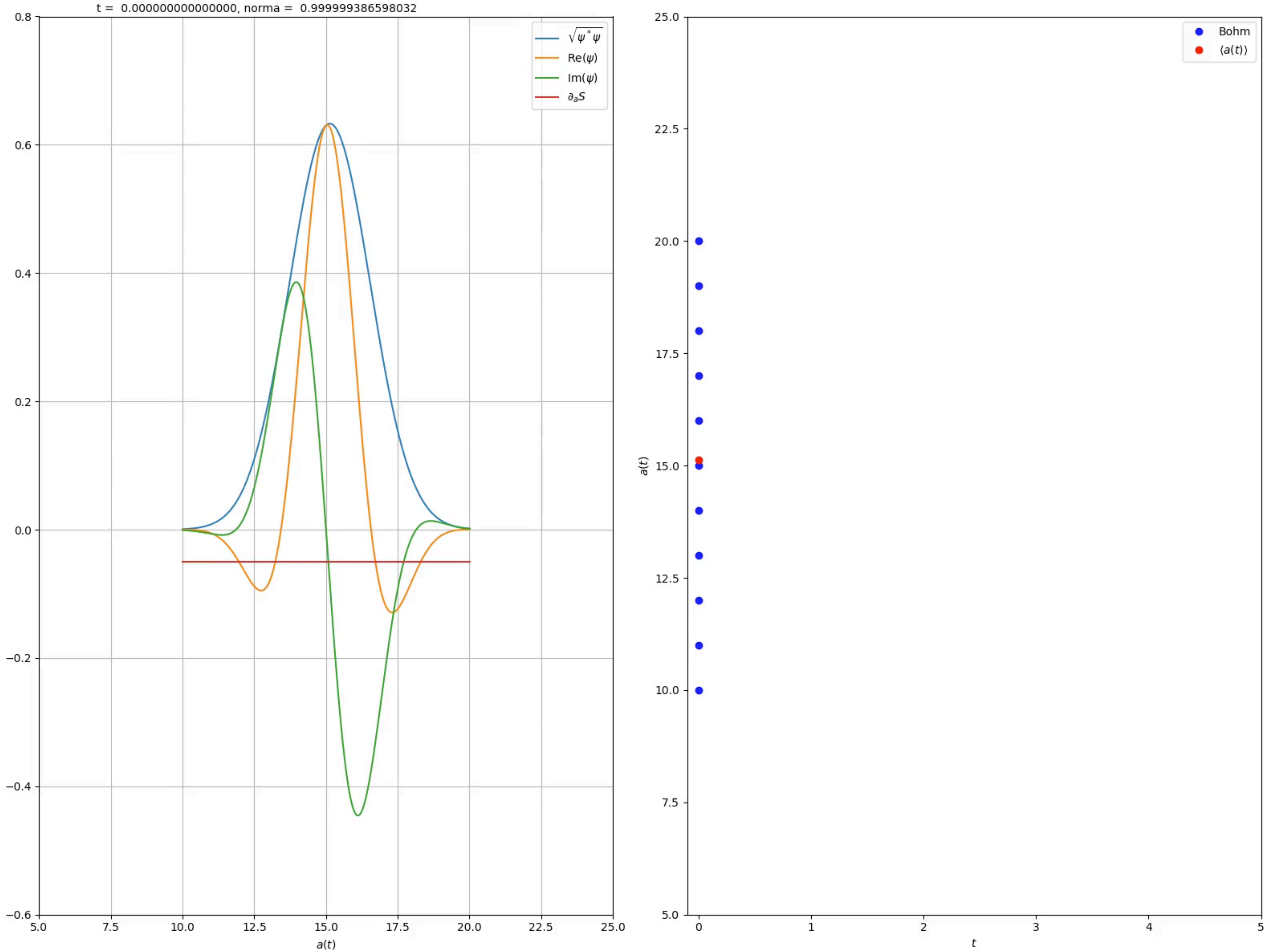




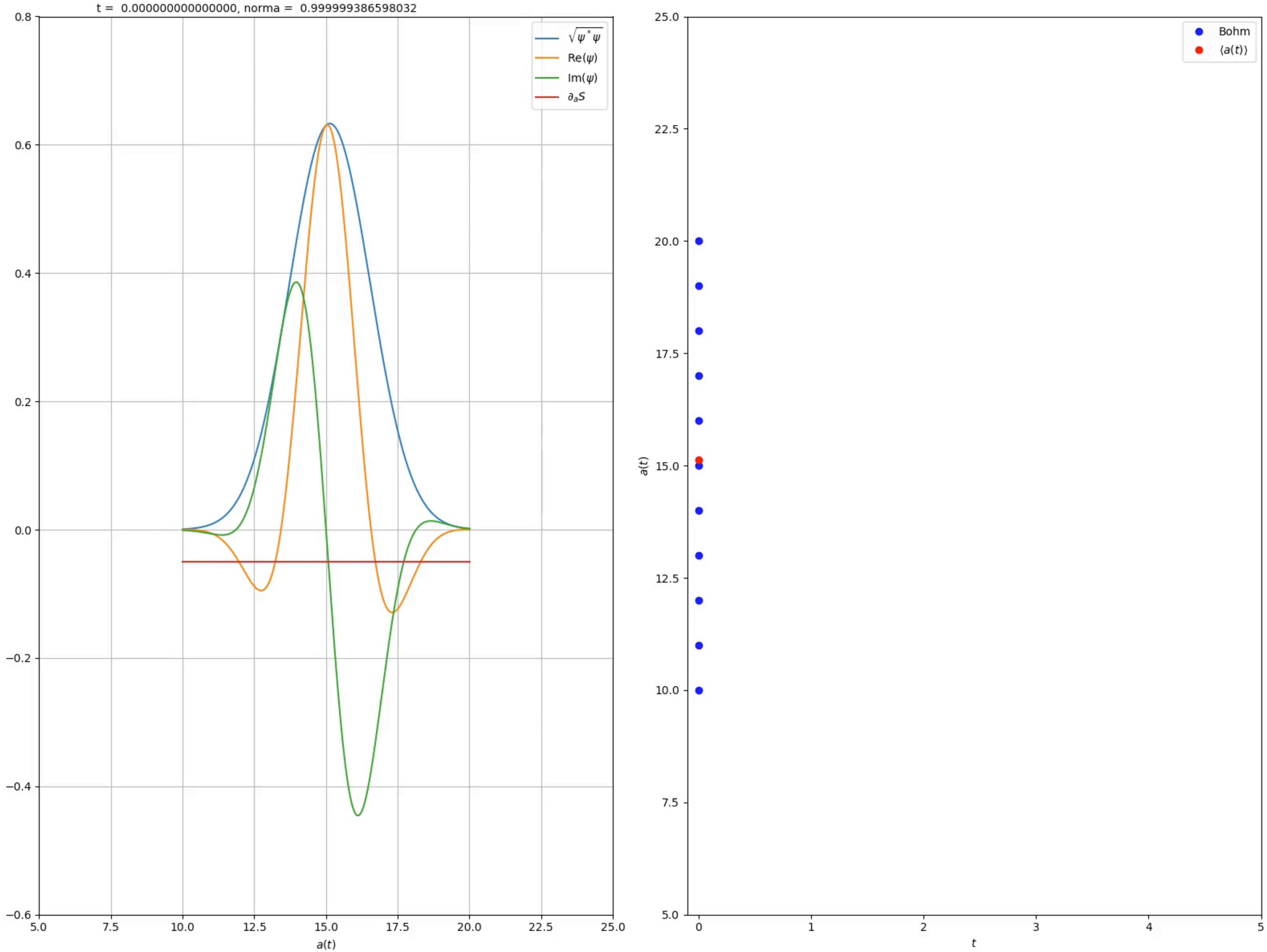








t = 0.000000000000000, norma = 0.999999386598032
$\sqrt{\psi^*\psi}$
Re(ψ)
Im(ψ)
$\partial_a S$
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
25.0
$a(t)$
Bohm
$\langle a(t) \rangle$
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
$a(t)$
0
1
2
3
4
5
t



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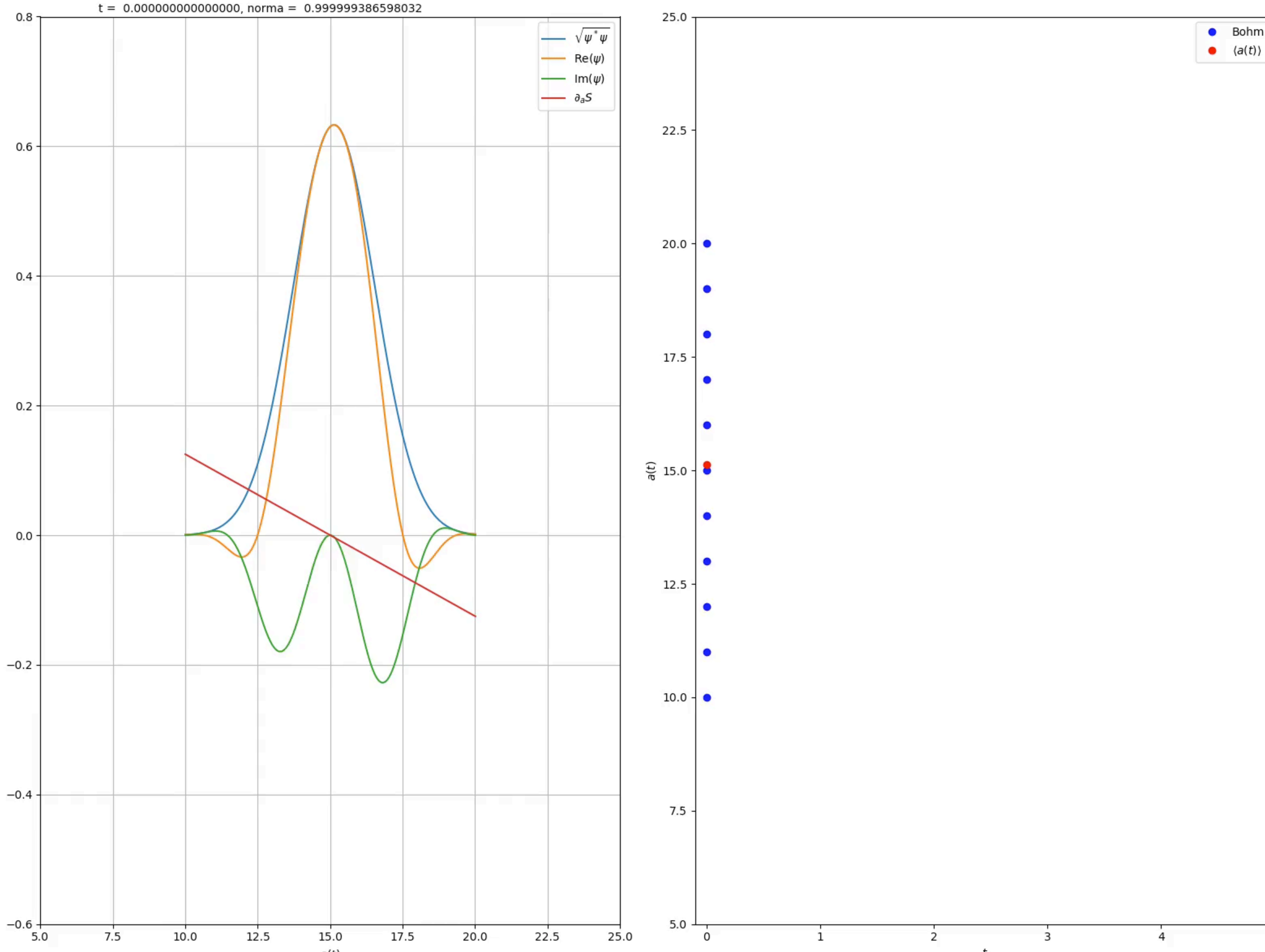
Bohm
$\langle a(t) \rangle$
$a(t)$
t



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$a(t)$
Bohm
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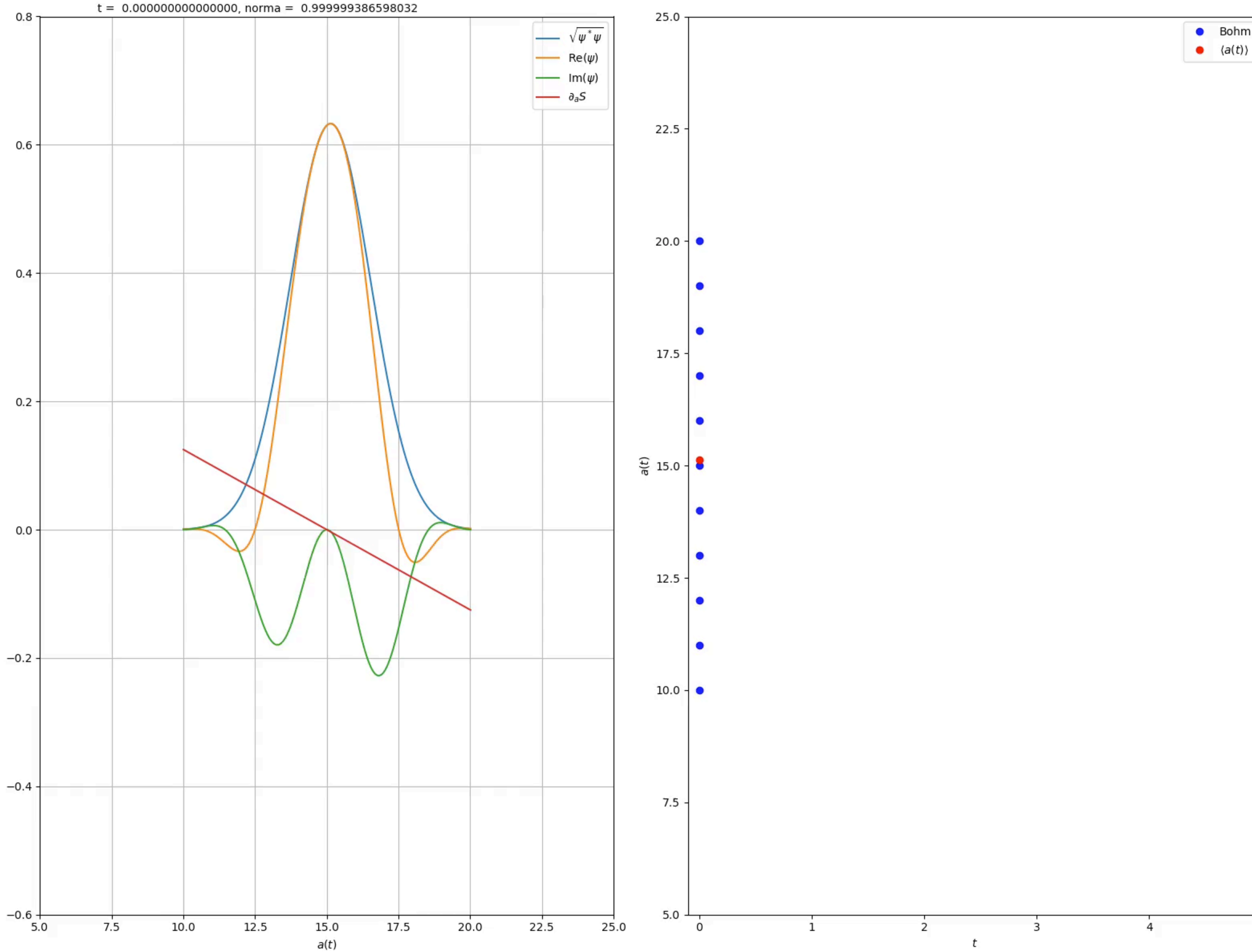
t



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$\langle a(t) \rangle$
$a(t)$
t

Quantum equilibrium

(Valentini & Westman, 2005)

$$i\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Particle in a box - 2D

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2\psi}{\partial y^2} + V\psi$$

V: infinite square well - size π

Density of actual configurations

$$\rho(x,y,t) \implies \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho\dot{x}) + \frac{\partial}{\partial y}(\rho\dot{y}) = 0$$

continuity equation

Energy eigenfunctions $\phi_{mn}(x,y) = \frac{2}{\pi}\sin(mx)\sin(ny)$

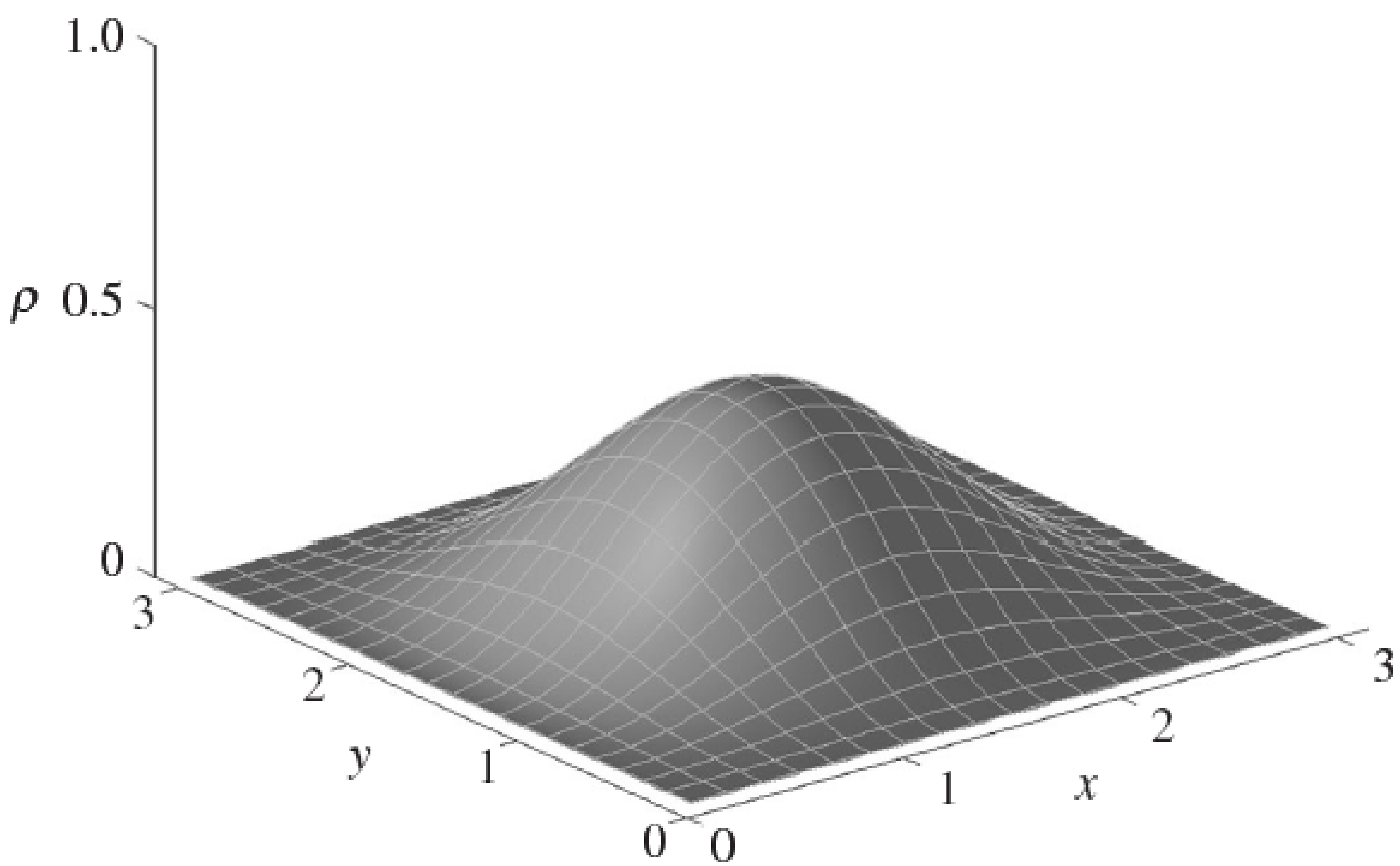
Energy levels $E_{mn} = \frac{1}{2}\left(m^2+n^2\right)$

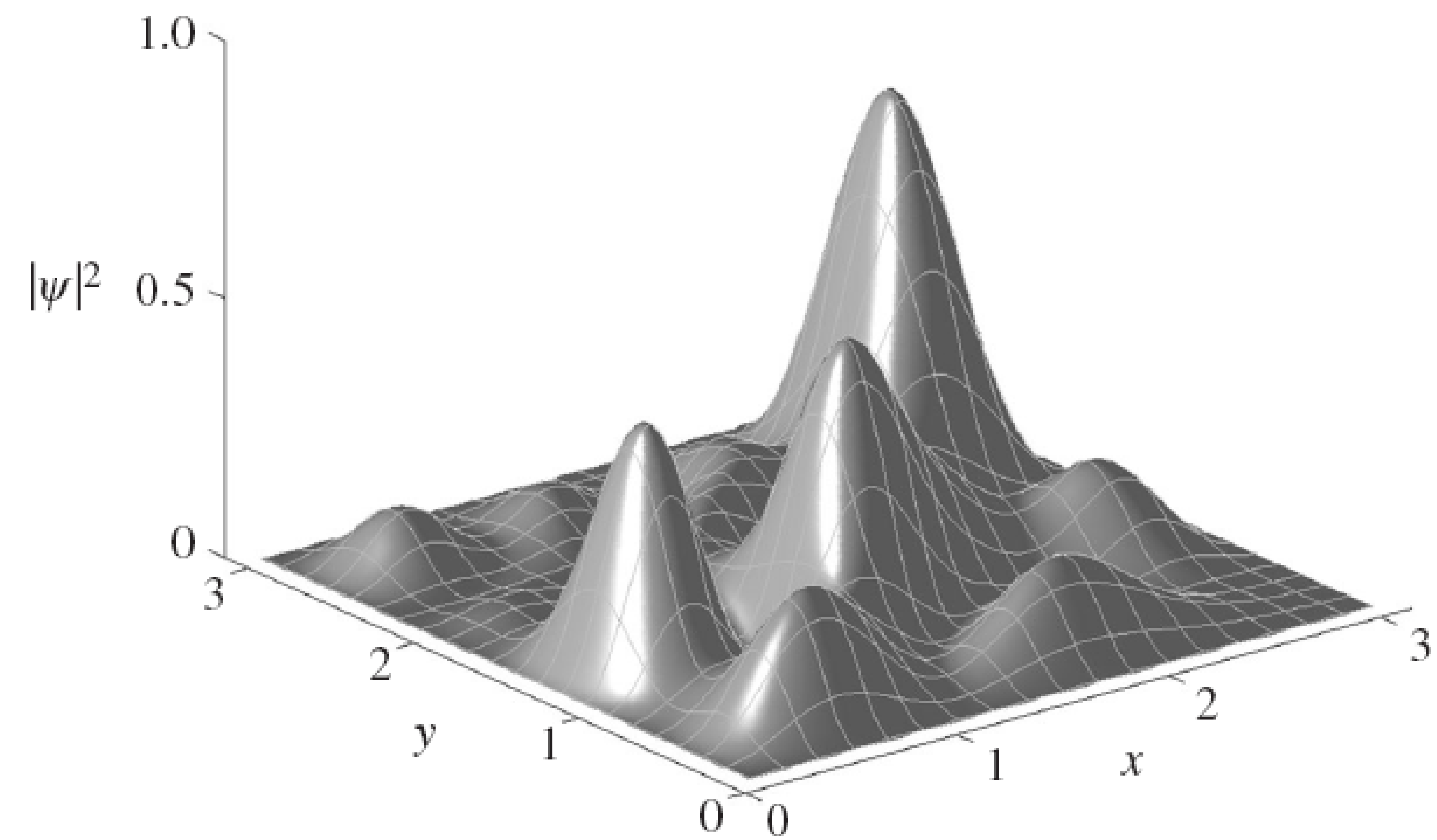
Initial configuration

$$\rho(x,y,0) = |\phi_{11}(x,y)|^2$$

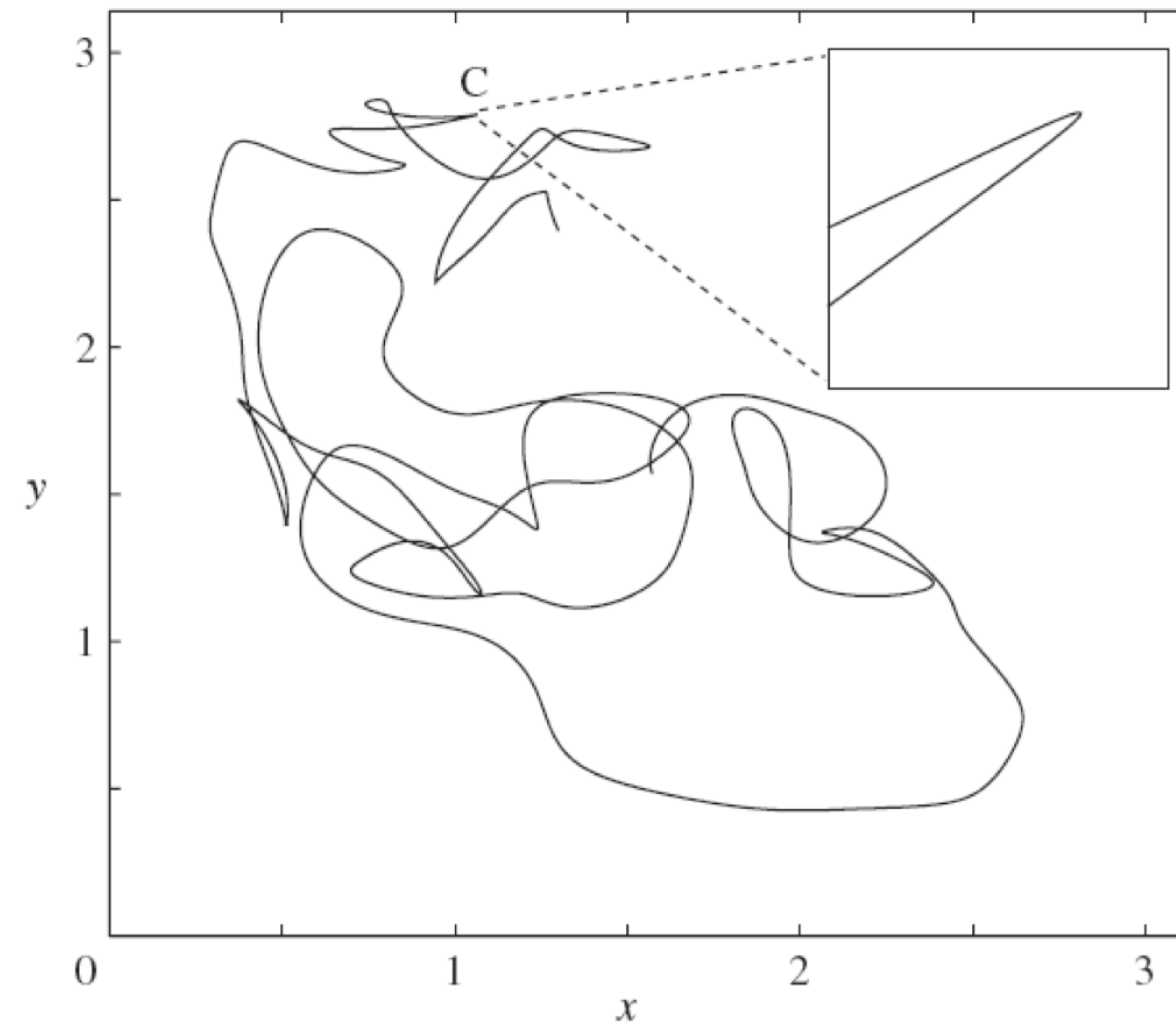
$$\psi(x,y,0) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp(i\theta_{mn})$$

$$\psi(x,y,t) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp i(\theta_{mn} - E_{mn}t)$$

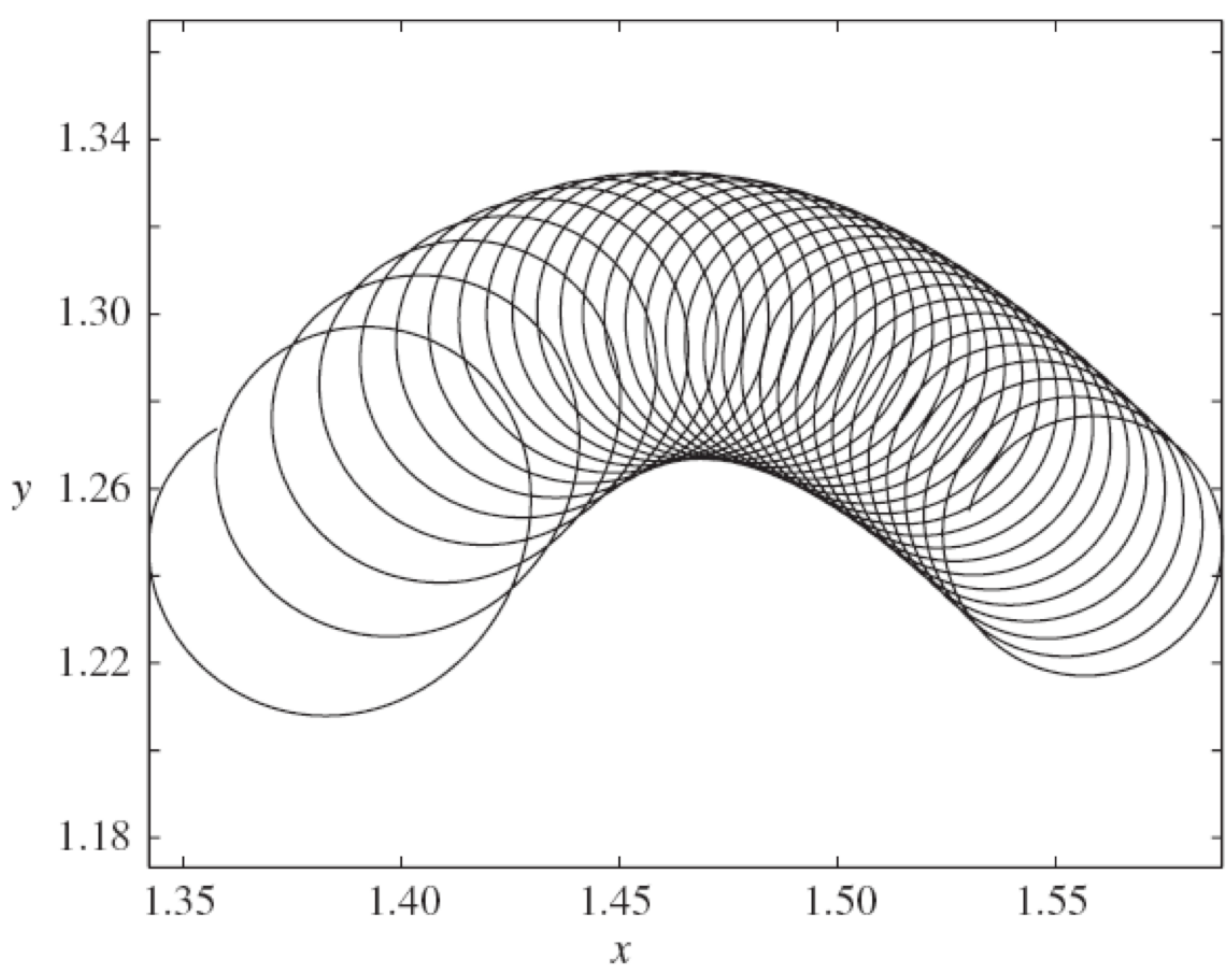


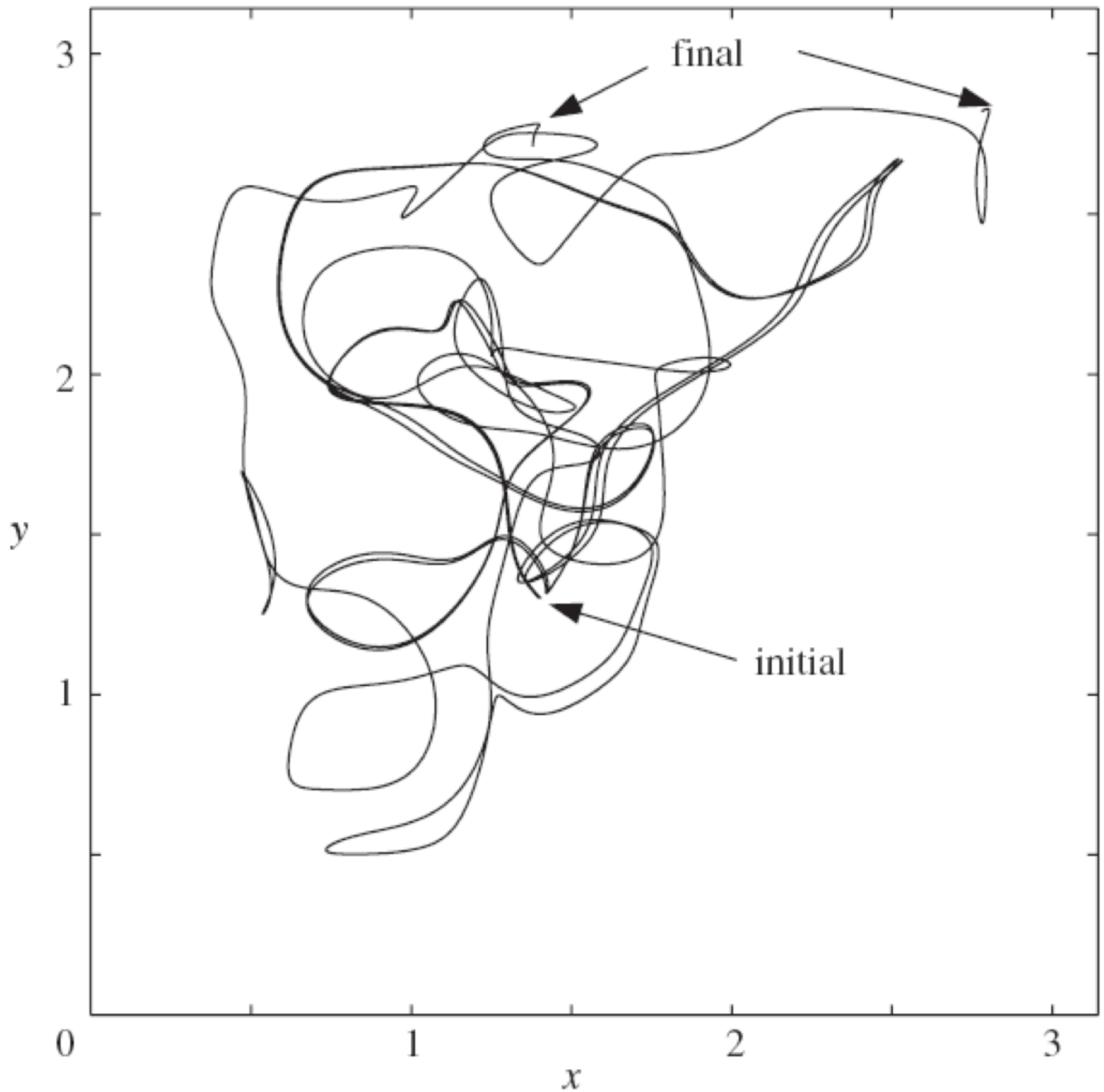


Typical quantum trajectory…



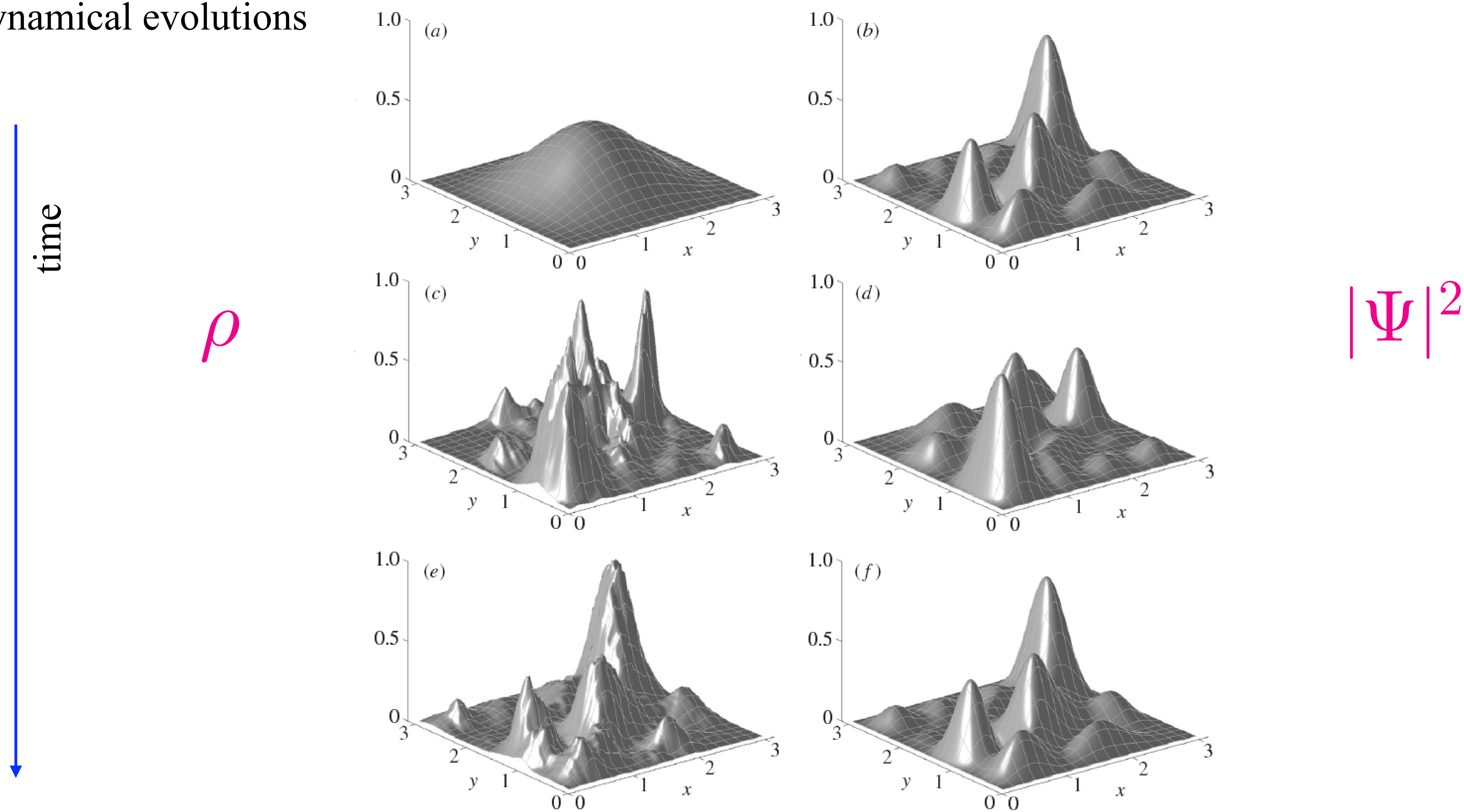
Close-up of a trajectory near a node

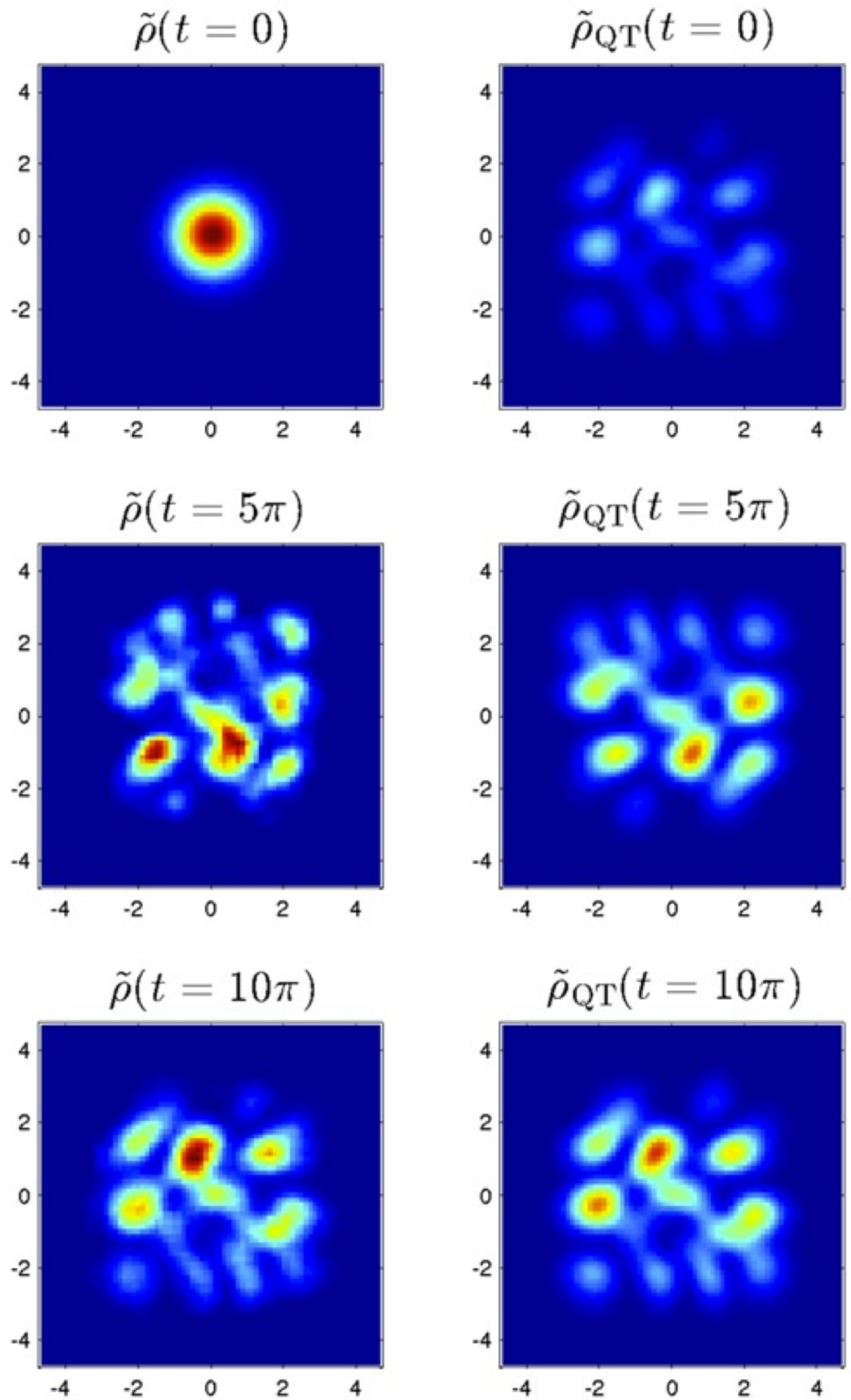




chaotic mixing…

Dynamical evolutions

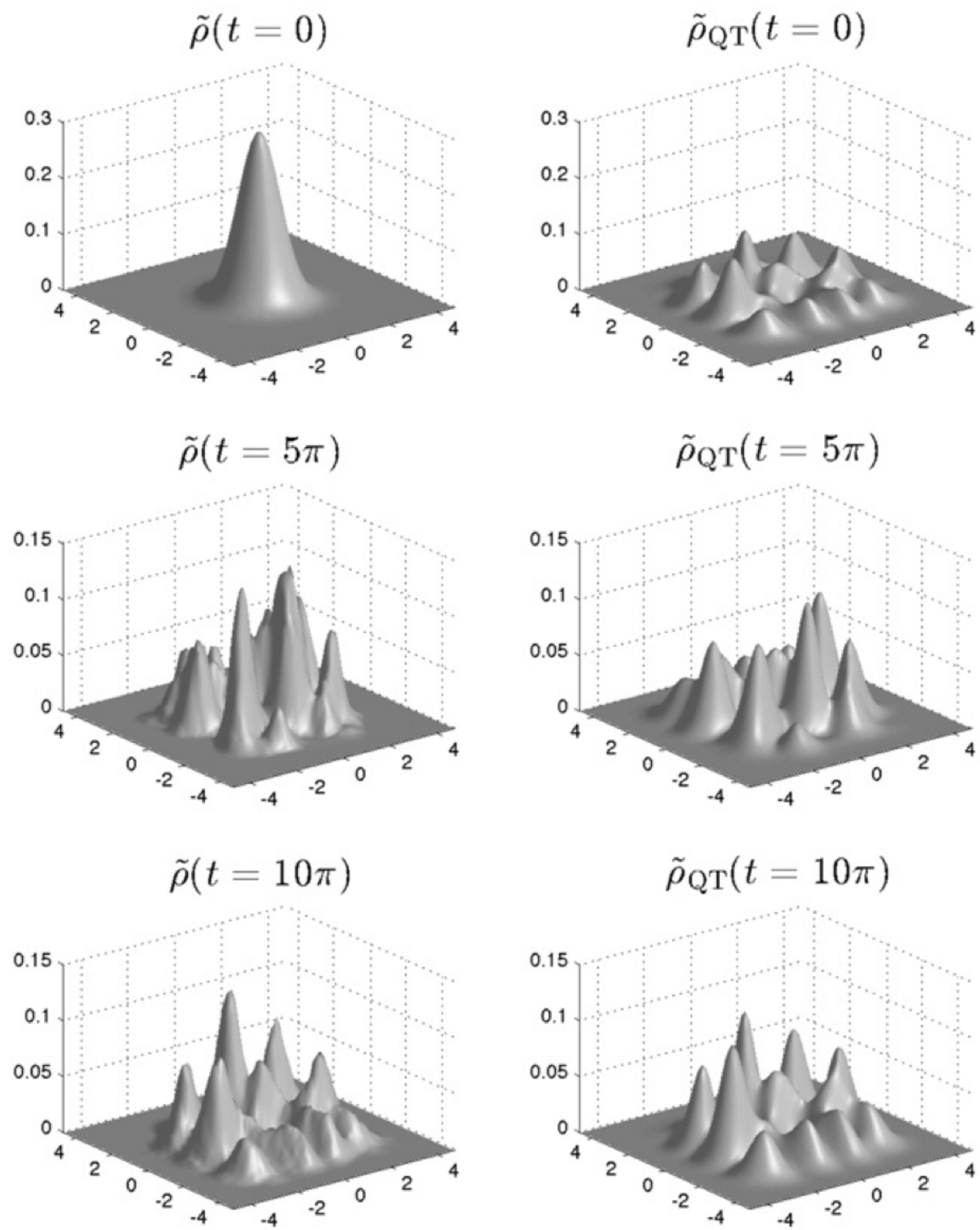




chaotic mixing…

relaxation towards equilibrium

just like ordinary thermal equilibrium



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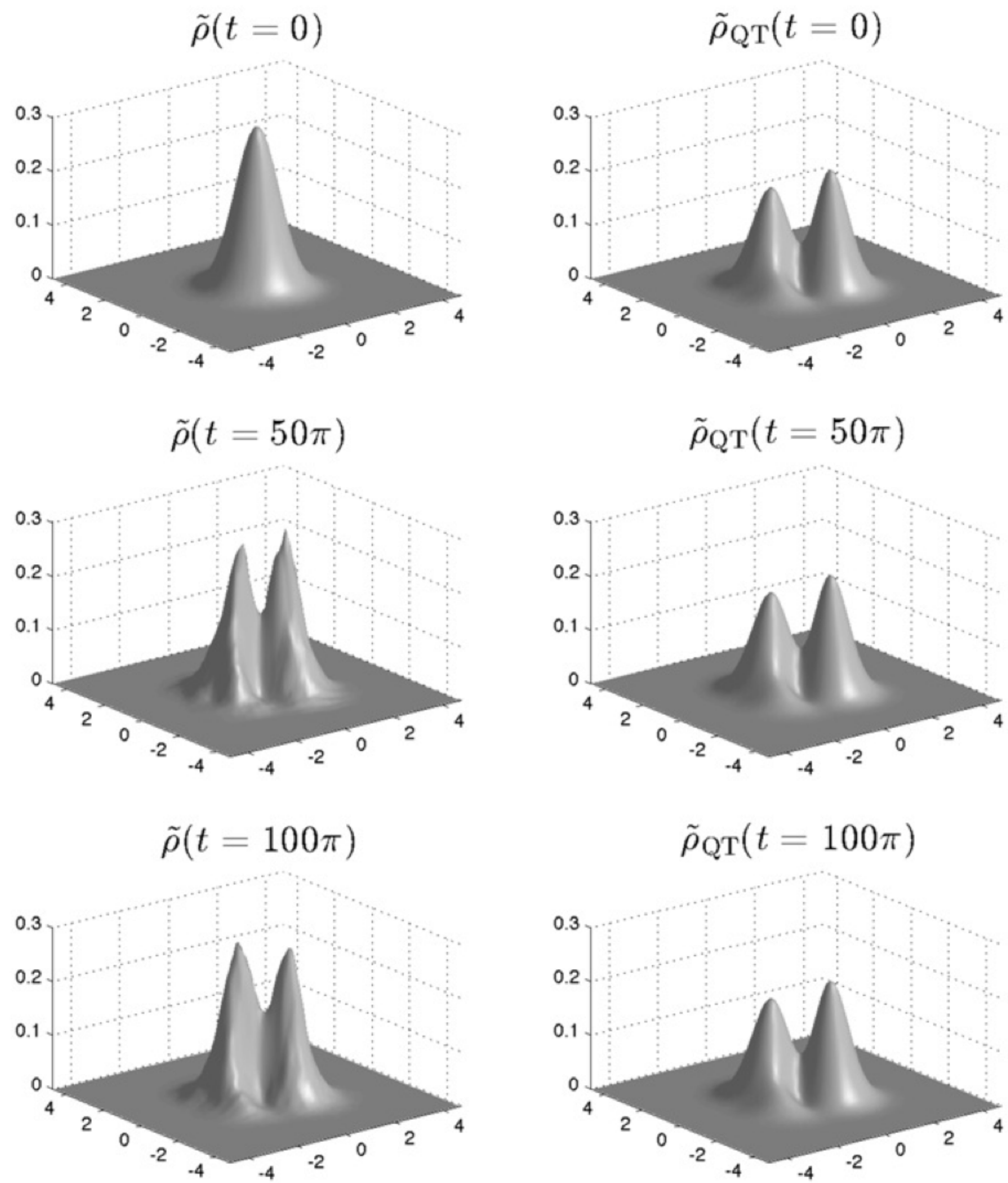
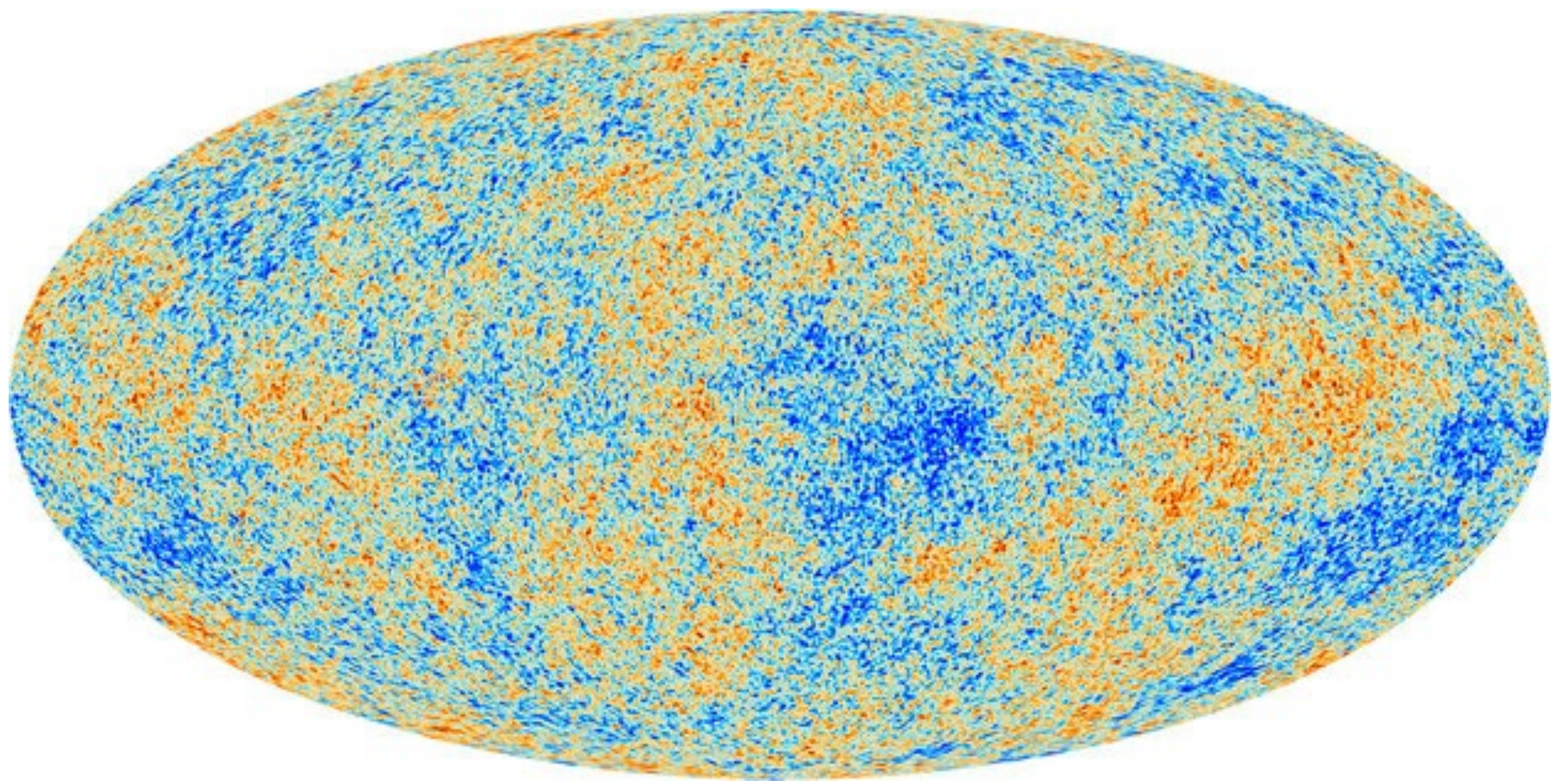


chaotic mixing…

relaxation towards equilibrium

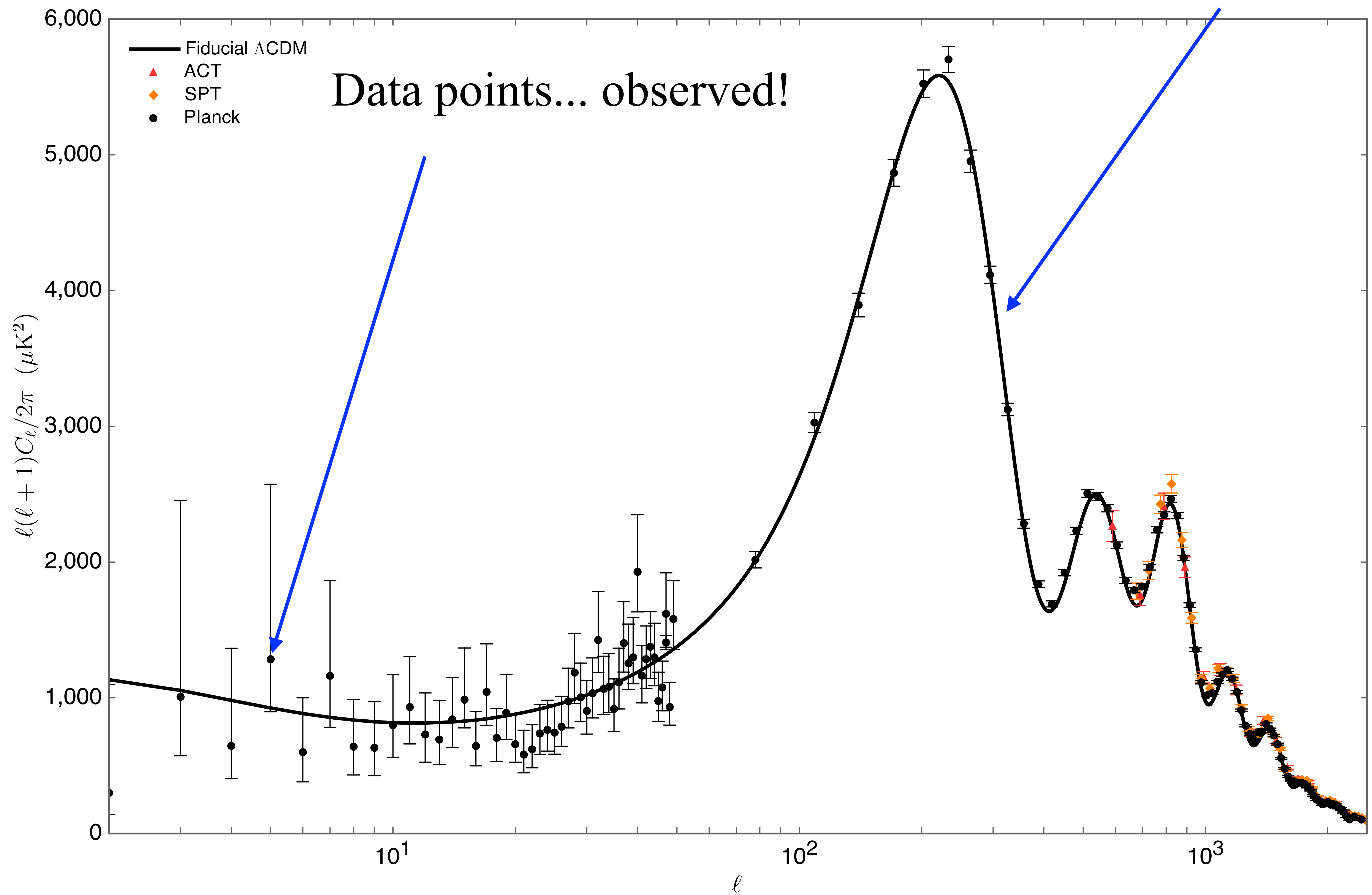
just like ordinary thermal equilibrium

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$



Back to cosmology

quantum vacuum fluctuations



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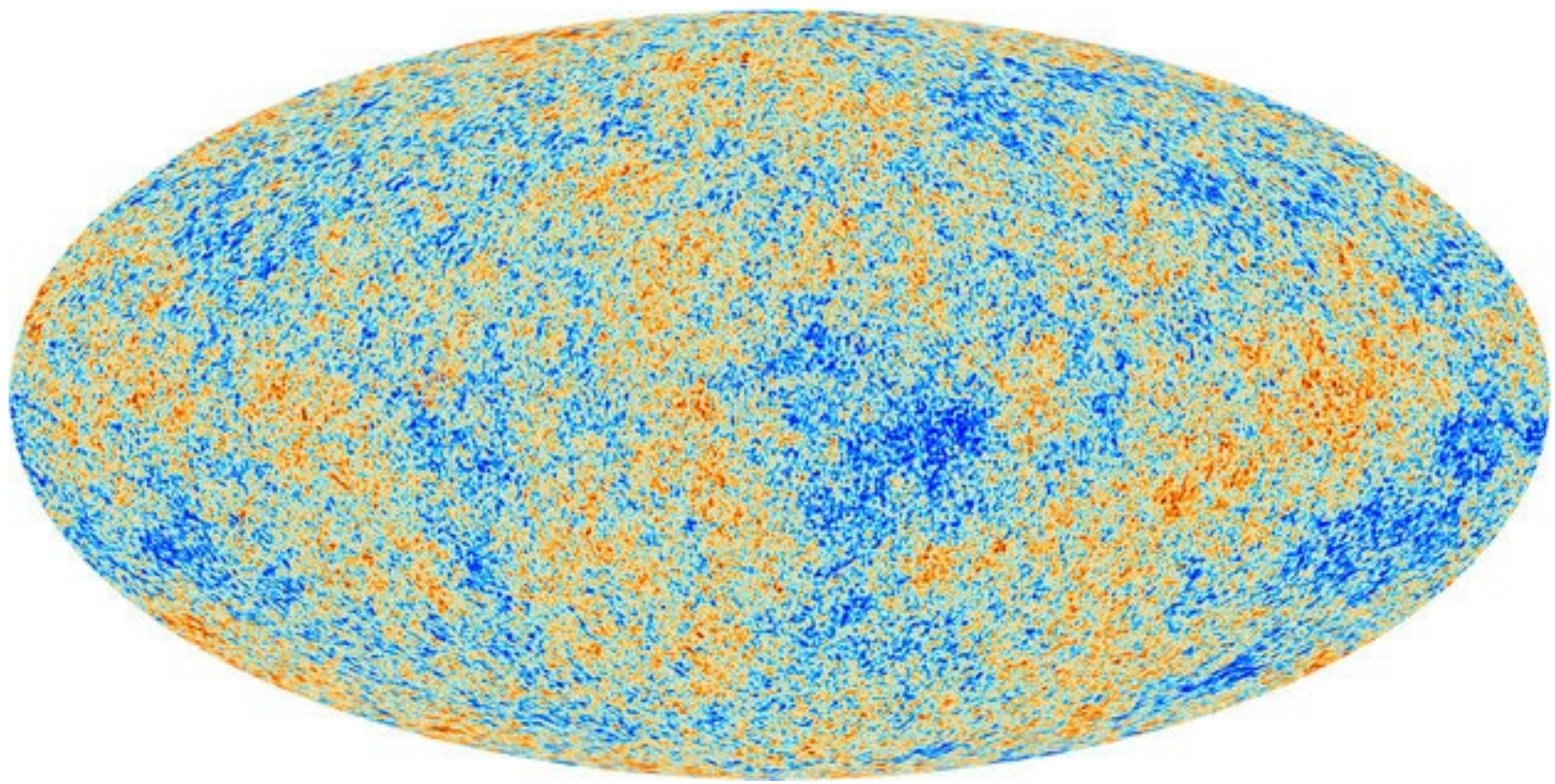
$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$



Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v \ \sim \Phi \sim \delta g_{00}$$

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v} \ \sim \Phi \sim \delta g_{00}$$

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

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second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4 x \left[\left(v'\right)^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2 \right]$

Inflationary perturbations: quantum fluctuations / expanding background

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$$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$$

slow-roll parameter

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

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variable-mass scalar field in Minkowski spacetime

$$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$$

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Inflationary perturbations: quantum fluctuations / expanding background

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variable-mass scalar field in Minkowski spacetime

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$

slow-roll parameter

+ Fourier transform $\quad v(\eta, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, v_{\boldsymbol{k}}(\eta) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Longrightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \left\{ v'_{\boldsymbol{k}} v^{*\prime}_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} - k^2\right]\right\}$$

Lagrangian formulation...

Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta,\boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

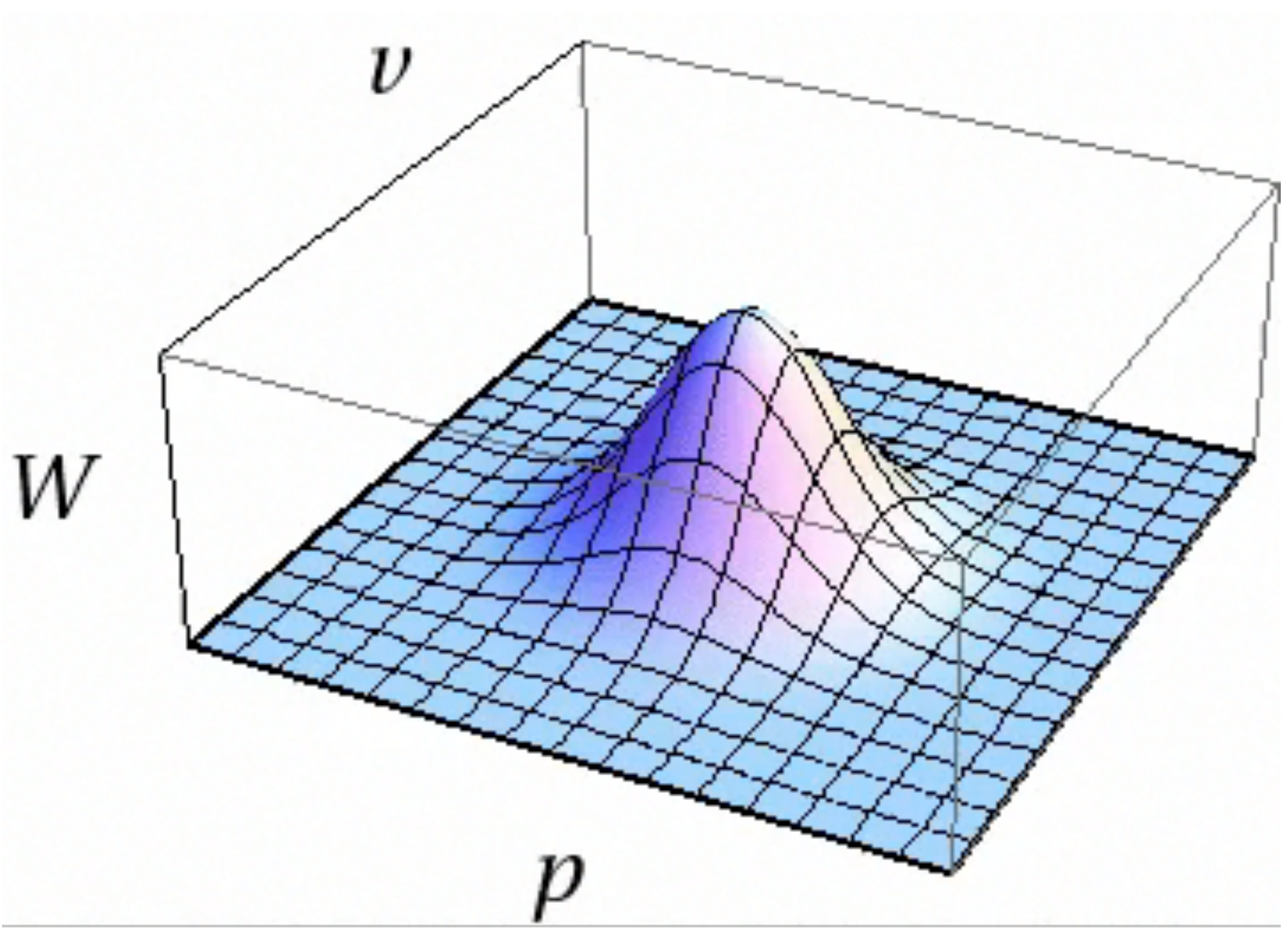
$$\Psi\left[v(\eta,\boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}, v_{\boldsymbol{k}}^{\mathrm{I}}\right) = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\mathrm{R}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}\right) \Psi_{\boldsymbol{k}}^{\mathrm{I}}\left(v_{\boldsymbol{k}}^{\mathrm{I}}\right)$$

$$\left.\begin{aligned} i\frac{\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}}{\partial\eta} &= \hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} \Psi_{\boldsymbol{k}}^{\mathrm{R,I}} \\ \hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} &= -\frac{1}{2}\frac{\partial^2}{\partial\left(v_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2} + \frac{1}{2}\omega^2(\eta,\boldsymbol{k})\left(\hat{v}_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2 \end{aligned}\right\}$$

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}\left(\eta\right)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right) = \int \frac{\mathrm{d}x}{2\pi^2} \Psi^*\left(v_{\boldsymbol{k}} - \frac{x}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}x} \Psi\left(v_{\boldsymbol{k}} + \frac{x}{2}\right)$

large squeezing limit ⇨ $W \propto \delta\left(p_{\boldsymbol{k}} + k \tan\phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

Ergodicity

realization, spatial direction

$$\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$$

Gaussian state solution $\Psi(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

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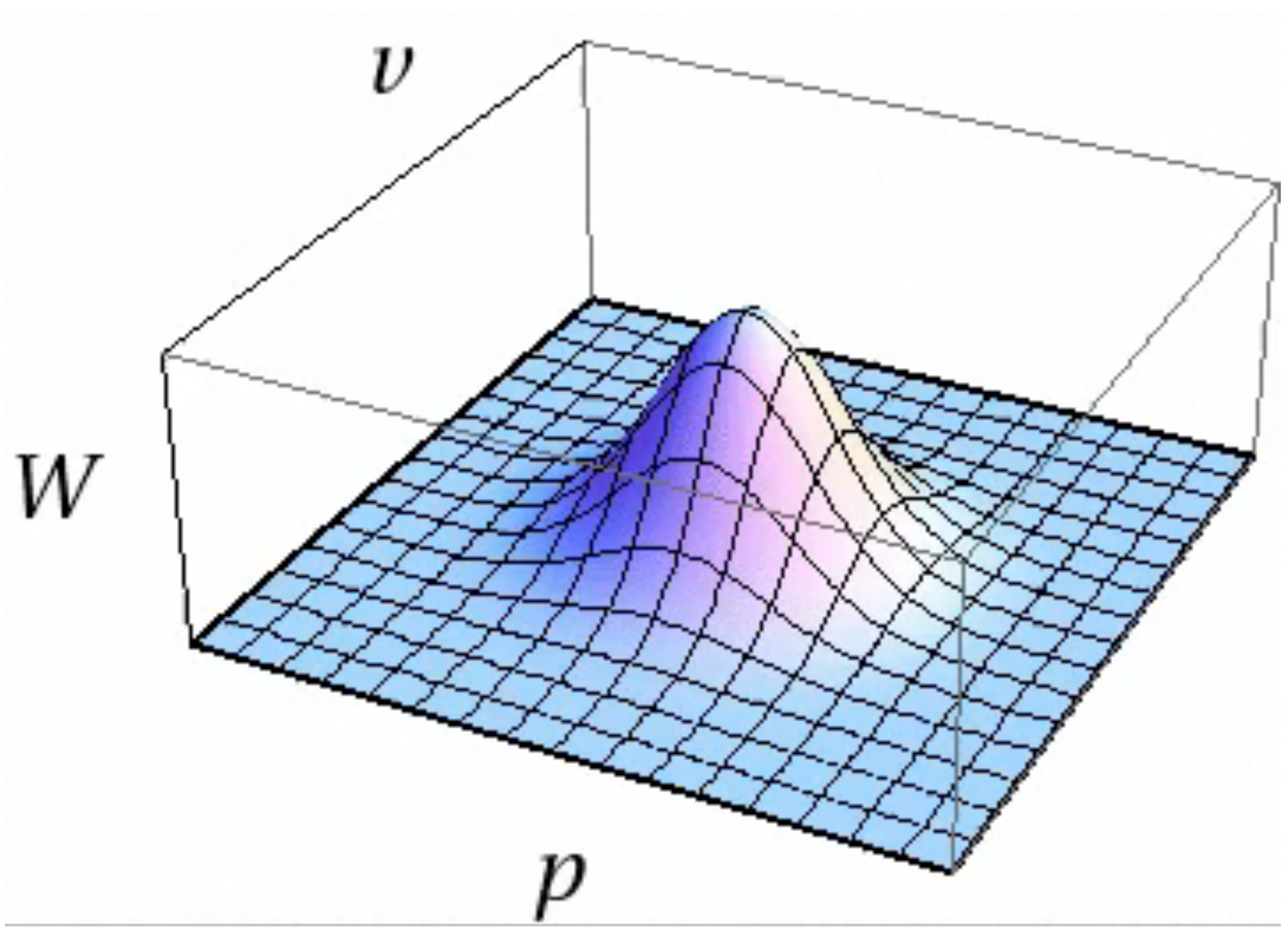


Stochastic distribution of classical processes

Ergodicity

realization — spatial direction

$$\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$$

Animation provided by V. Vennin

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

Power-law inflation example

with $$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and $$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

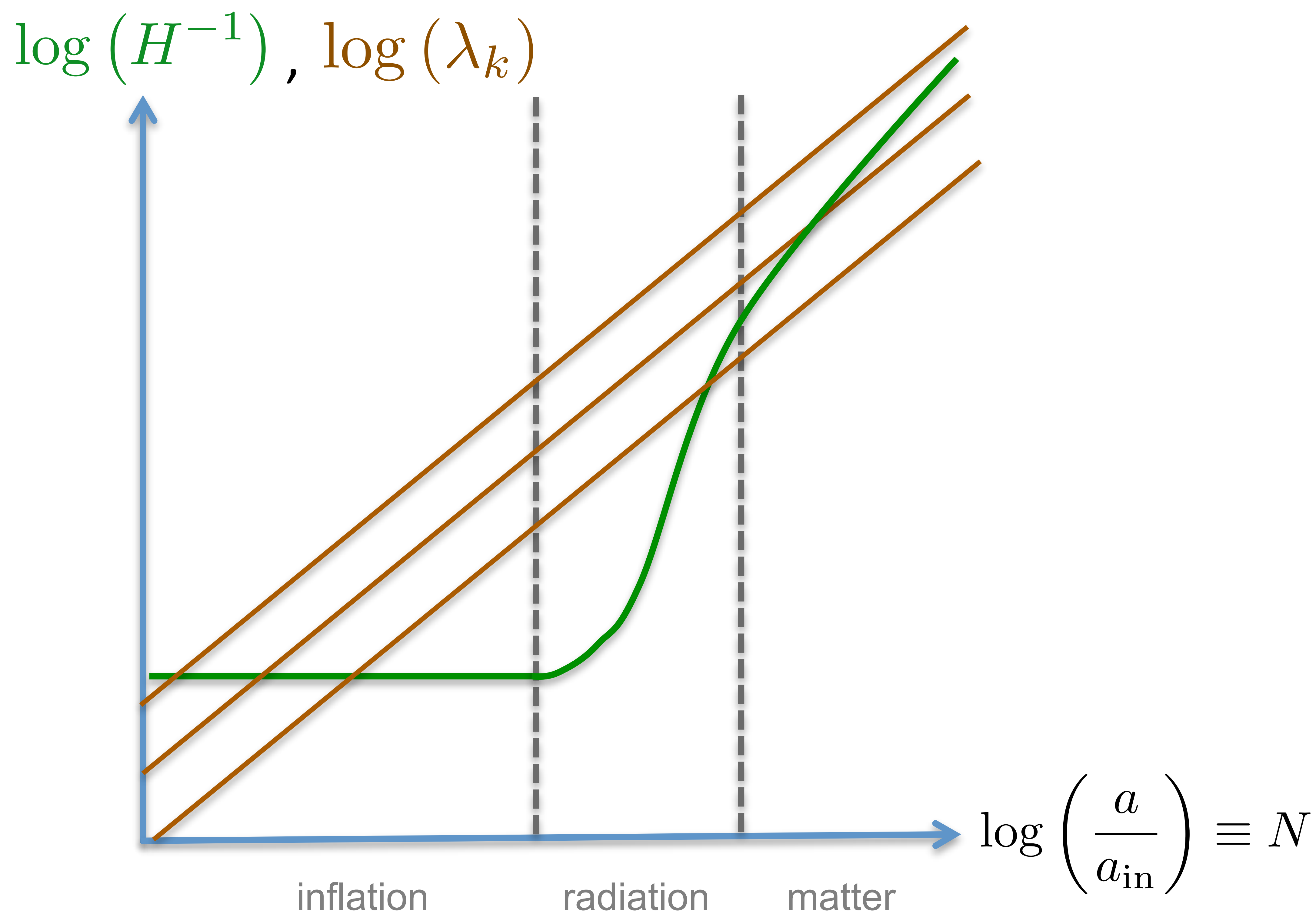
with

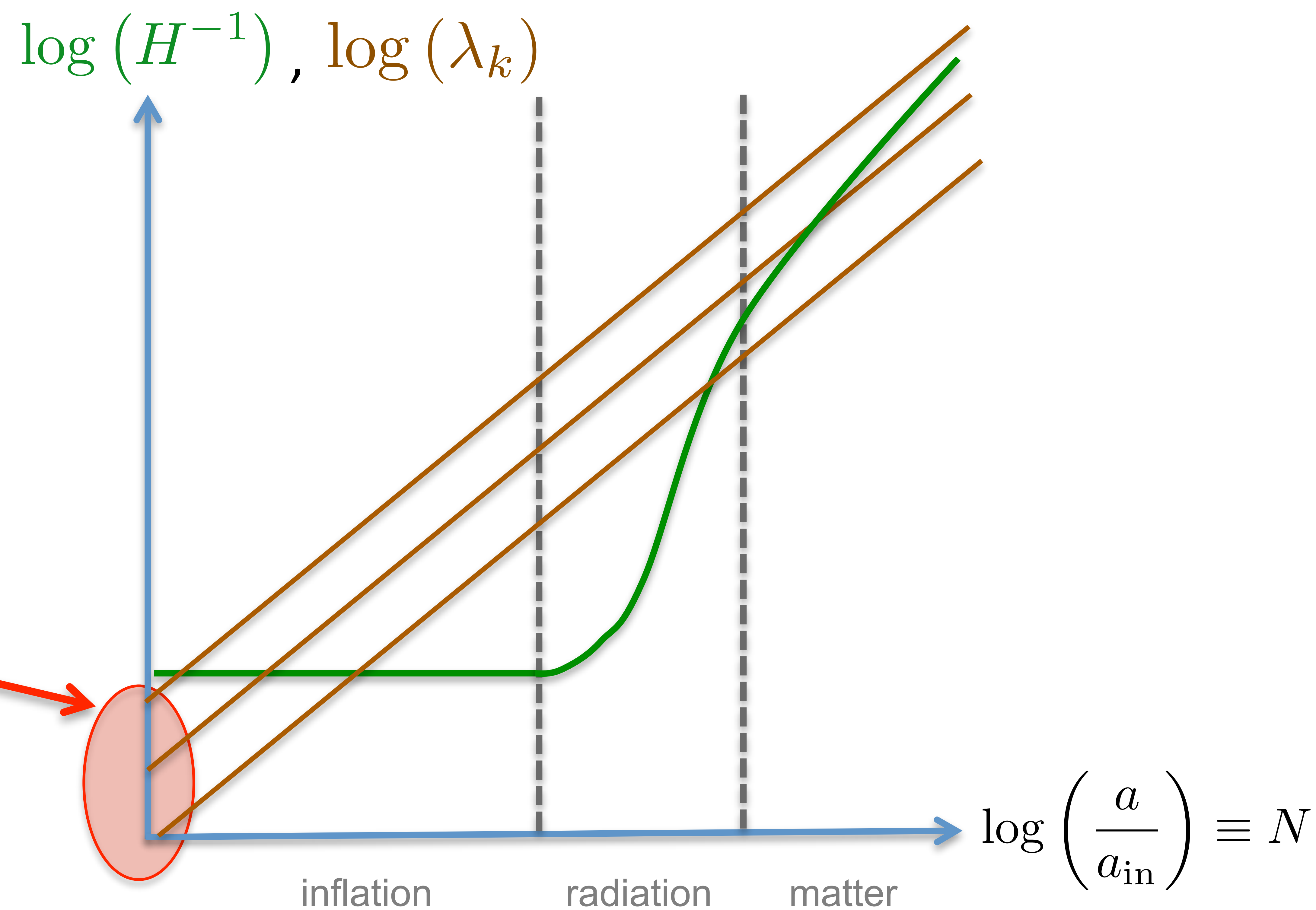
$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k}, \eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega'_{\boldsymbol{k}} = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k})$$

$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f'_{\boldsymbol{k}}}{f_{\boldsymbol{k}}}$$

$$f''_{\boldsymbol{k}} + \omega^2(\boldsymbol{k}, \eta) f_{\boldsymbol{k}} = 0$$


$\log\left(H^{-1}\right)$, $\log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter



Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

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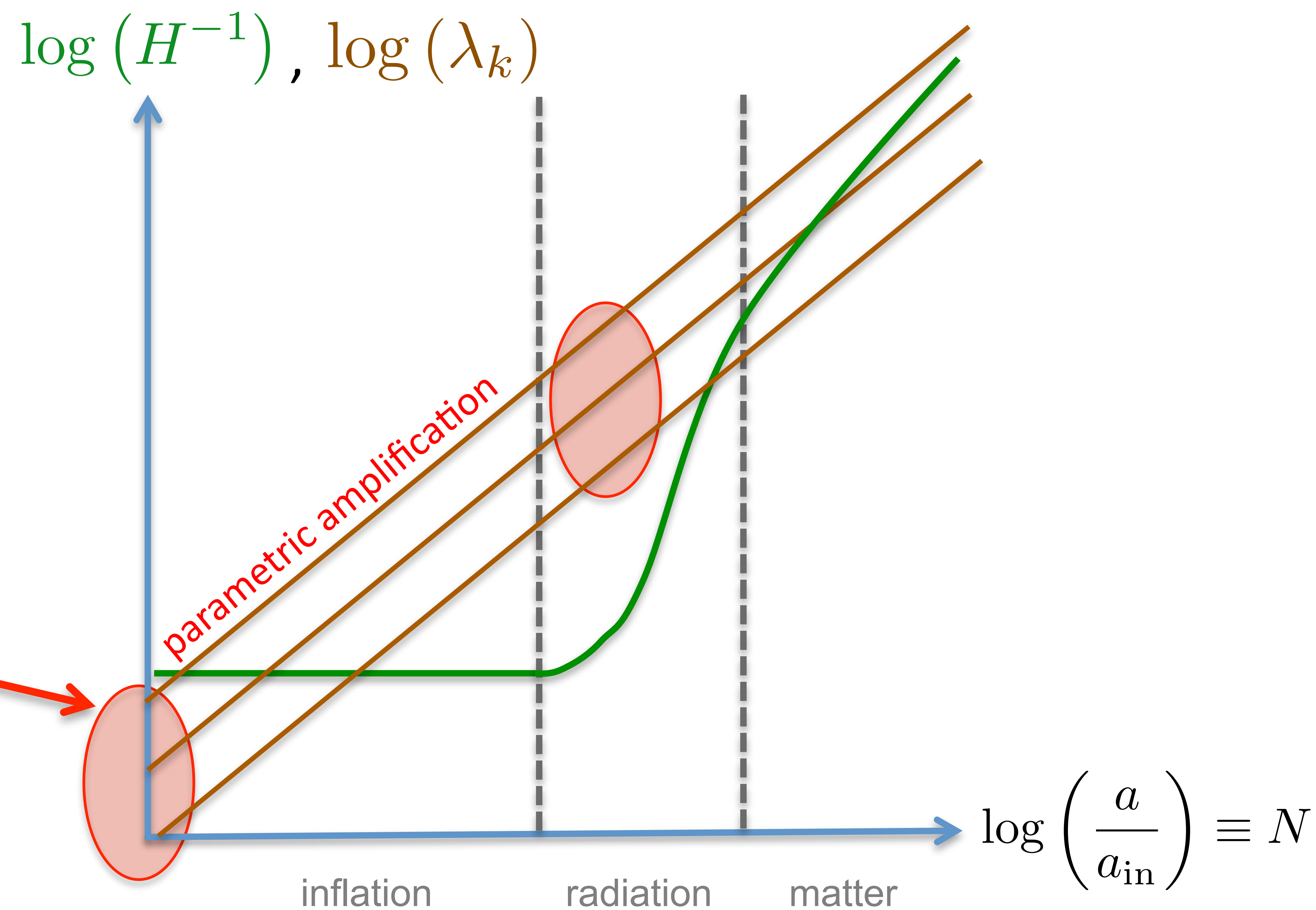

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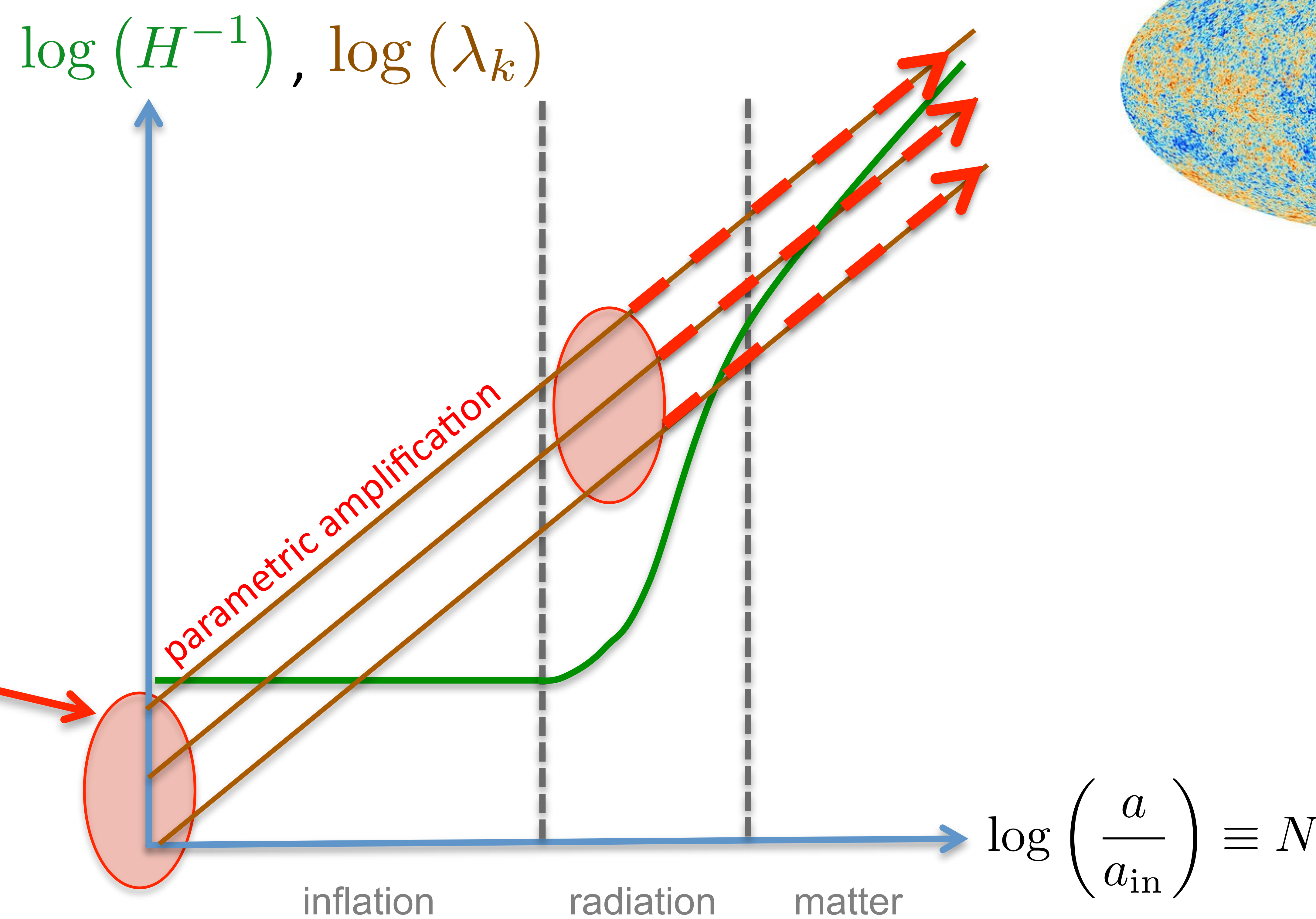
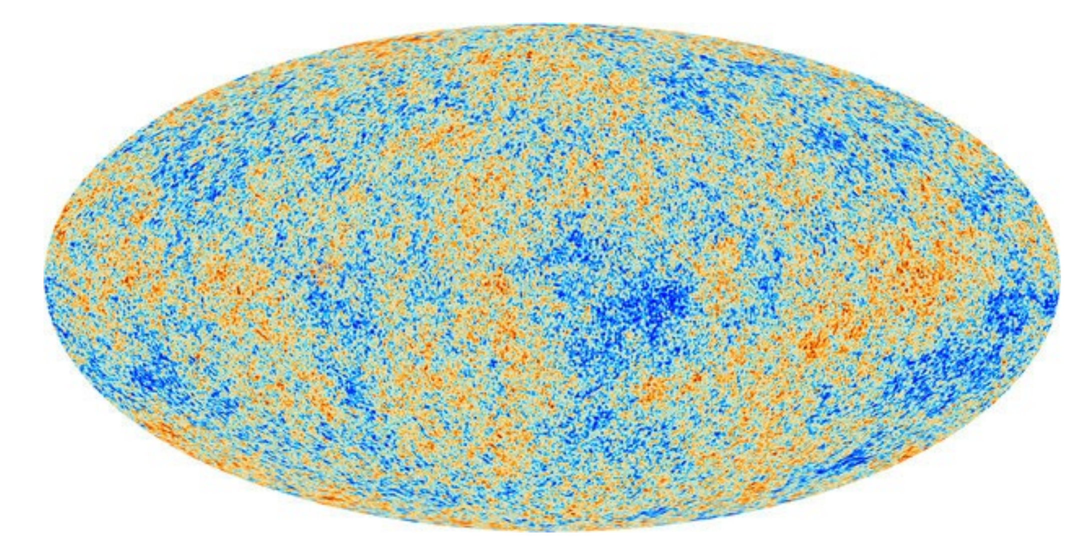




Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta \sim -2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta \sim -2}{\simeq} \frac{\ell_0}{-k\eta}$

Sub-Hubble regime

$\lambda \ll H^{-1}$

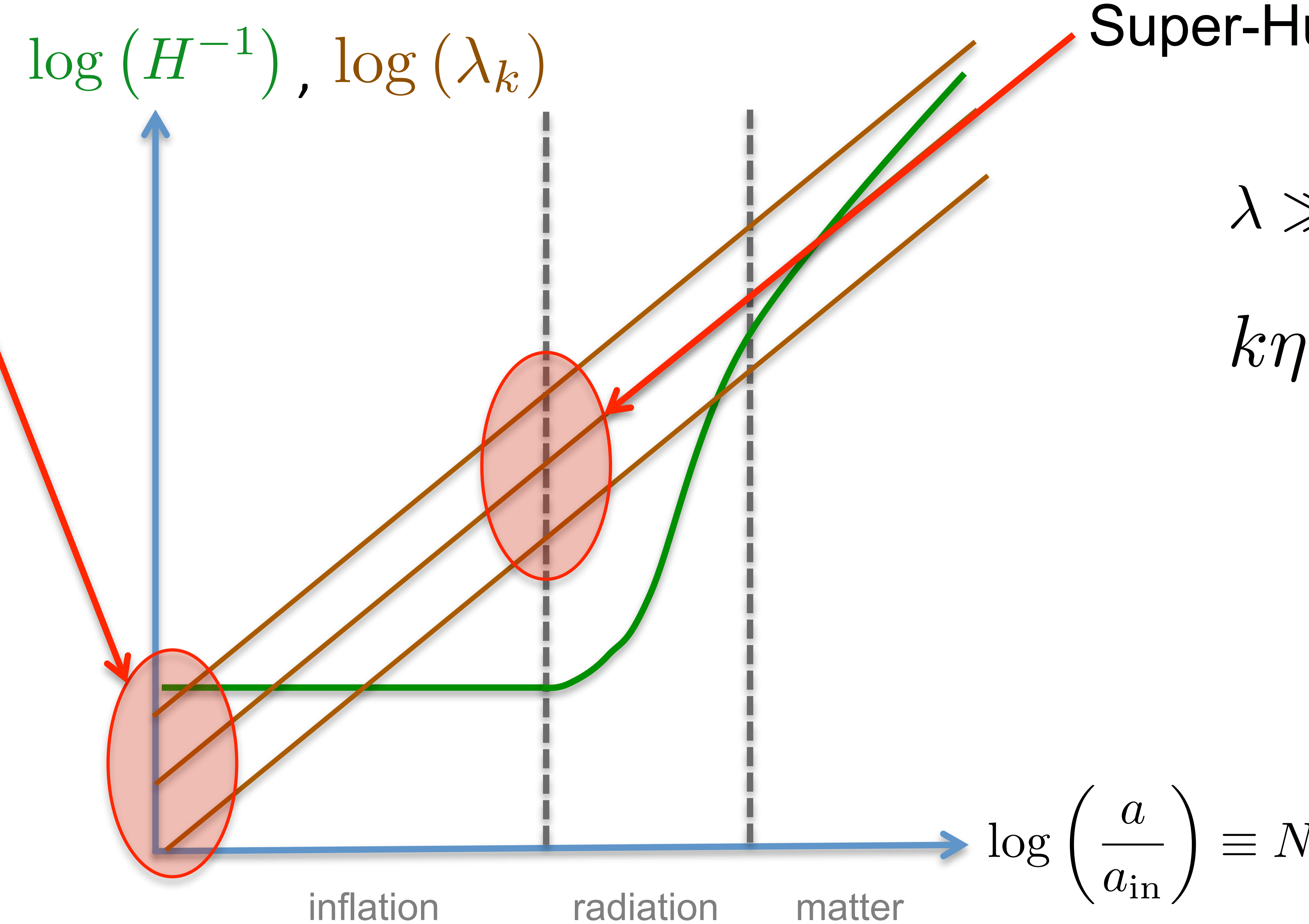
$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

$$v_{\boldsymbol{k}}'' + \left[\boldsymbol{k}^2 - U(\eta)\right] v_{\boldsymbol{k}} = 0$$

Vacuum state

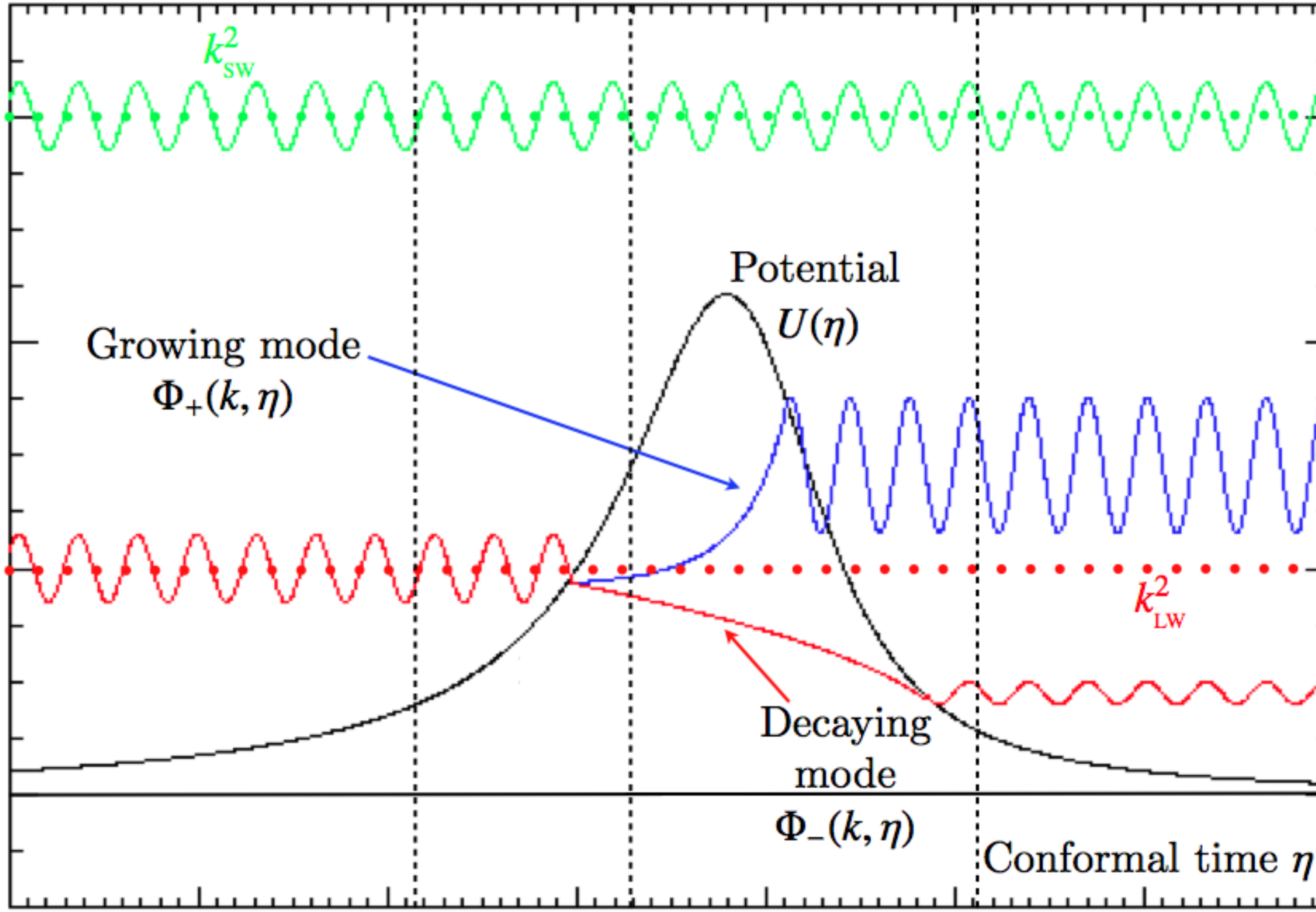
$$v_{\boldsymbol{k}} \underset{|k\eta|\to\infty}{\longrightarrow} \frac{e^{-ik\eta}}{\sqrt{2k}}$$

Initial conditions fixed!

compare

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - U(x)\right]\Psi = 0$$

(time independent
Schrödinger equation)



Transmission & Reflexion coefficients!

$$v''_{\boldsymbol{k}} + \left[\boldsymbol{k}^2 - U(\eta)\right] v_{\boldsymbol{k}} = 0$$

Vacuum state

$$v_{\boldsymbol{k}} \underset{|k\eta|\to\infty}{\longrightarrow} \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}}$$

Initial conditions fixed!

compare

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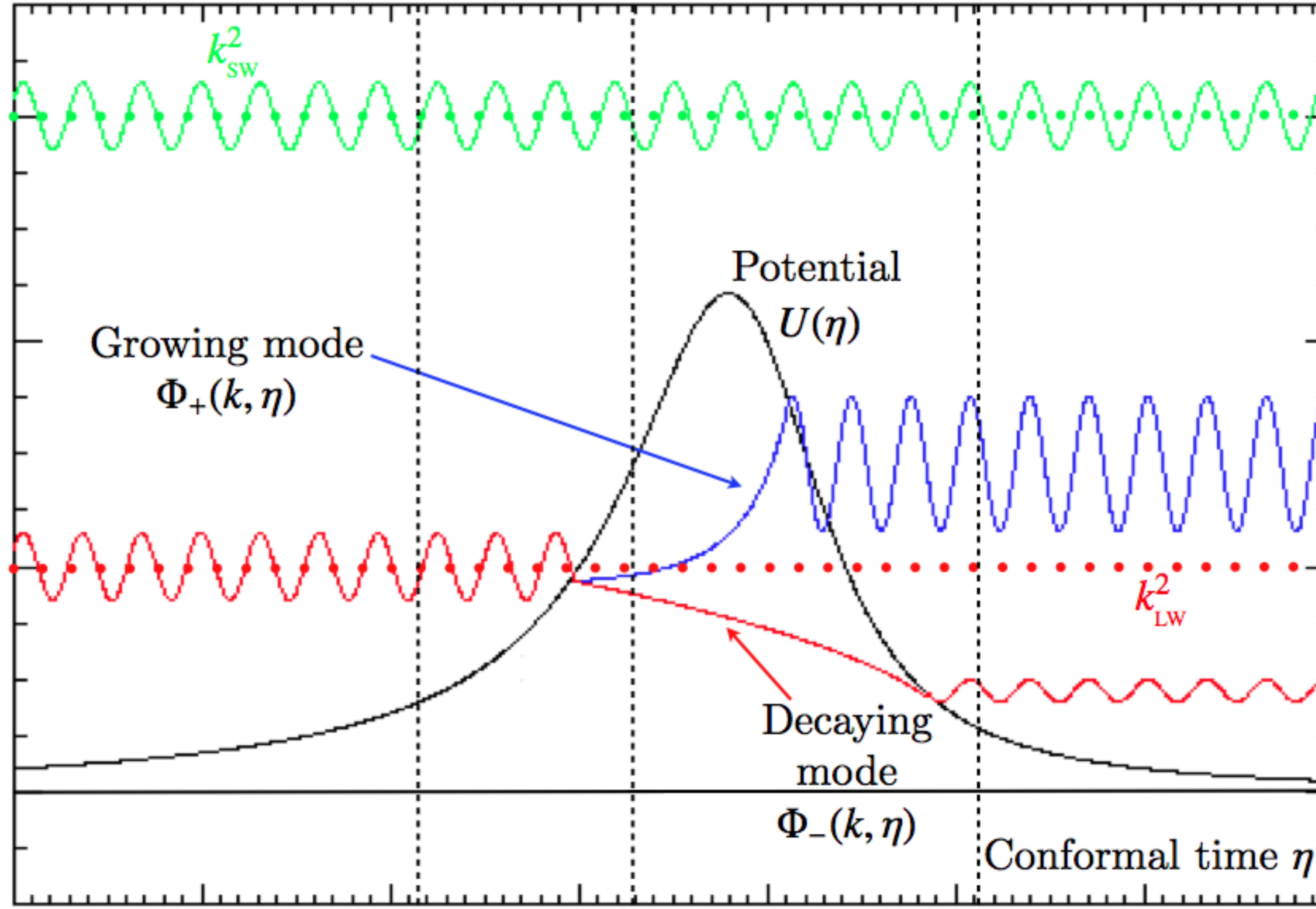


→ Transmission & Reflexion coefficients!

Primordial Power Spectrum

Standard case

$$\boxed{f''_{\boldsymbol{k}} + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

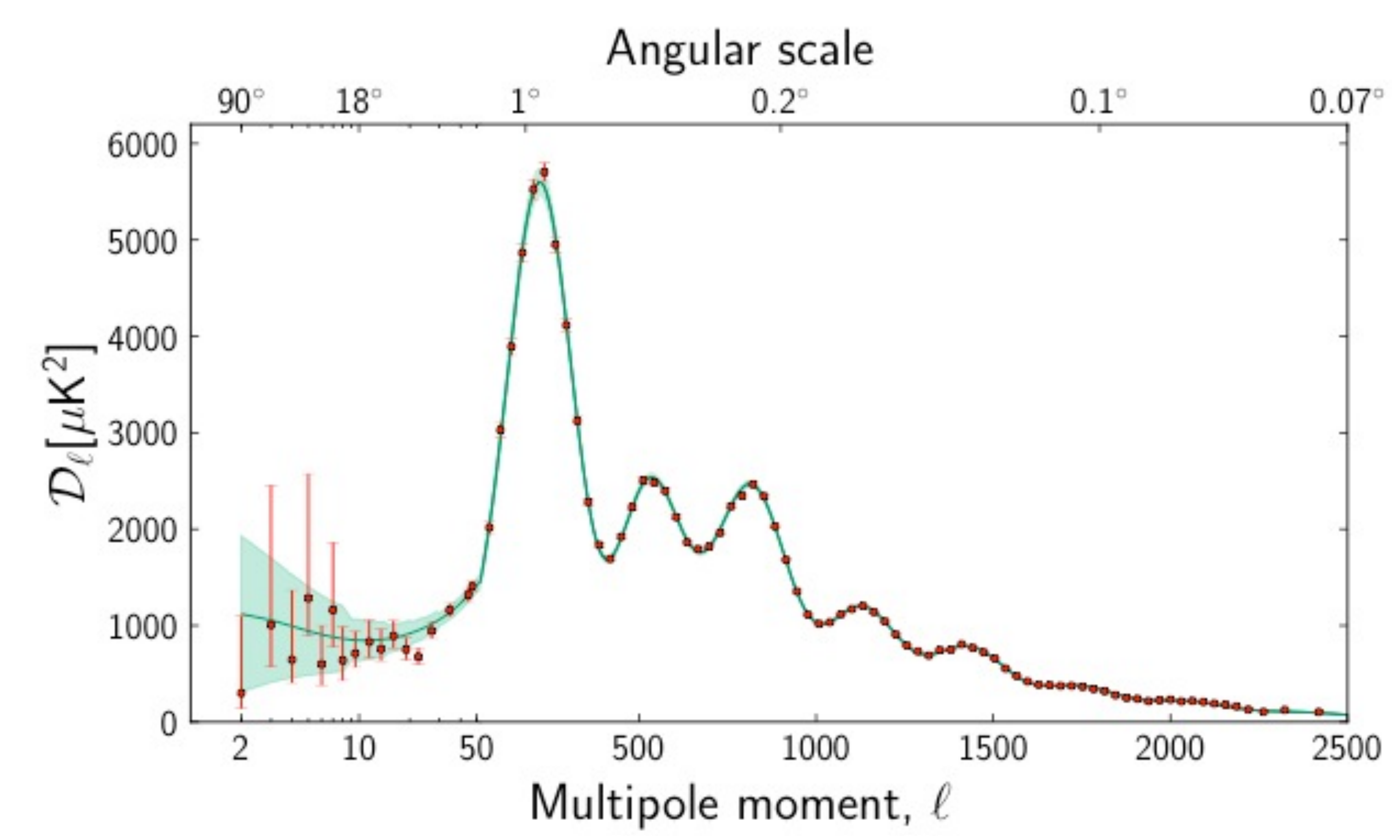
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f'_{\boldsymbol{k}}}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}^2_{\boldsymbol{k}} \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation$(k\eta \to 0^-)$, this gives $P_v(k) = \frac{k^3}{2\pi^3}\left(\langle \hat{v}^2_{\boldsymbol{k}} \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \frac{1}{2a^2 M^2_{\mathrm{Pl}} \epsilon_1} P_v(k) = A_S k^{n_{\mathrm{S}}-1}$

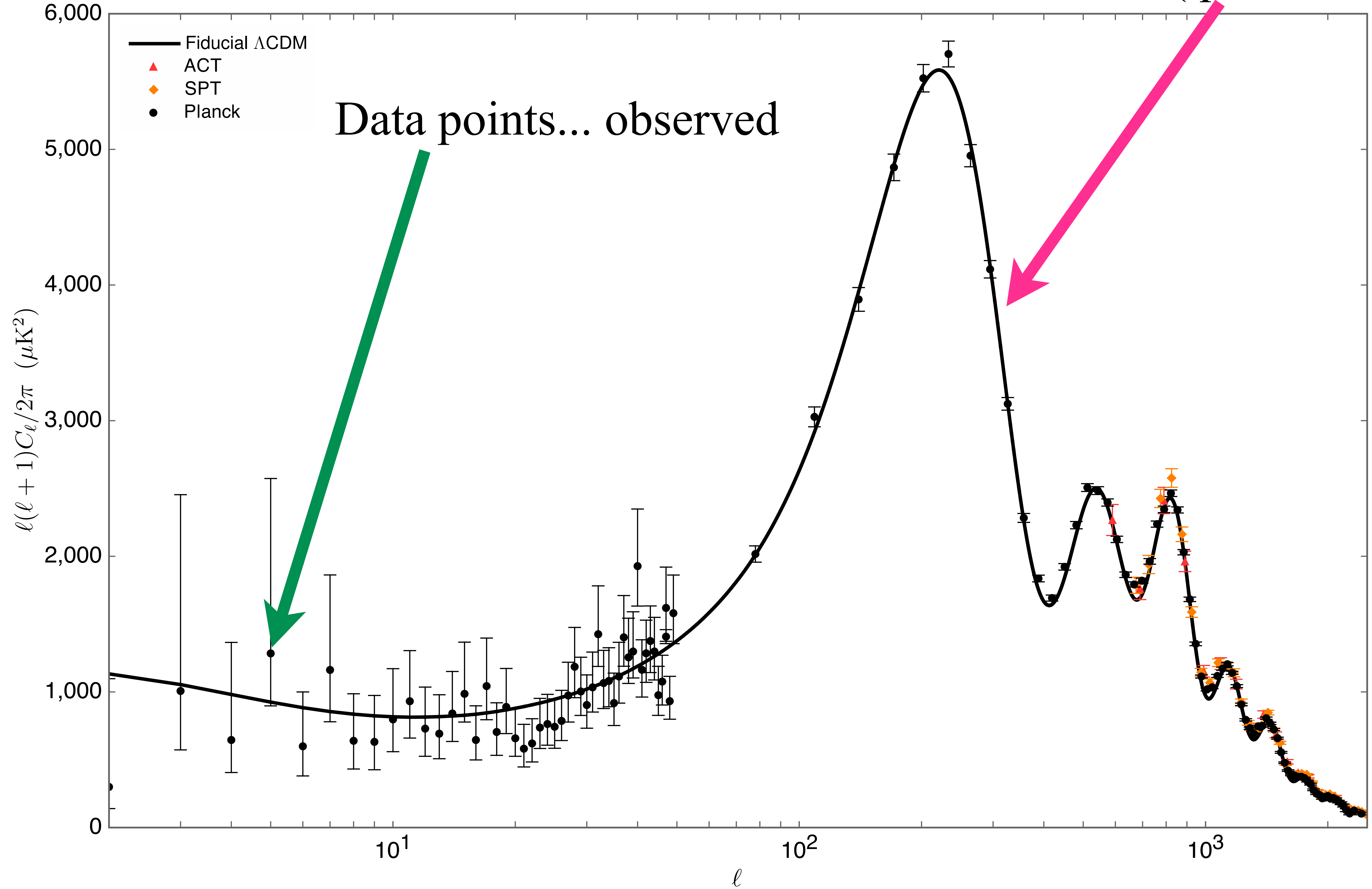
with $n_{\mathrm{S}} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_{\mathrm{s}} = 0.0389 \pm 0.0054$



Planck + ACT + SPT data

Theoretical prediction
(quantum vacuum fluctuations)



Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order

$$\mathcal{S}_{\mathrm{E-H}} = \int \mathrm{d}^4 x \left[R^{(0)} + \delta^{(2)} R\right]$$

Self-consistent treatment of the perturbations?

Hamiltonian up to 2nd order

$$H = H_{(0)} + H_{(2)} + \cdots$$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\, a \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a, T\right) \Psi_{(2)}\left[v, T; a\left(T\right)\right]$$

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Classical

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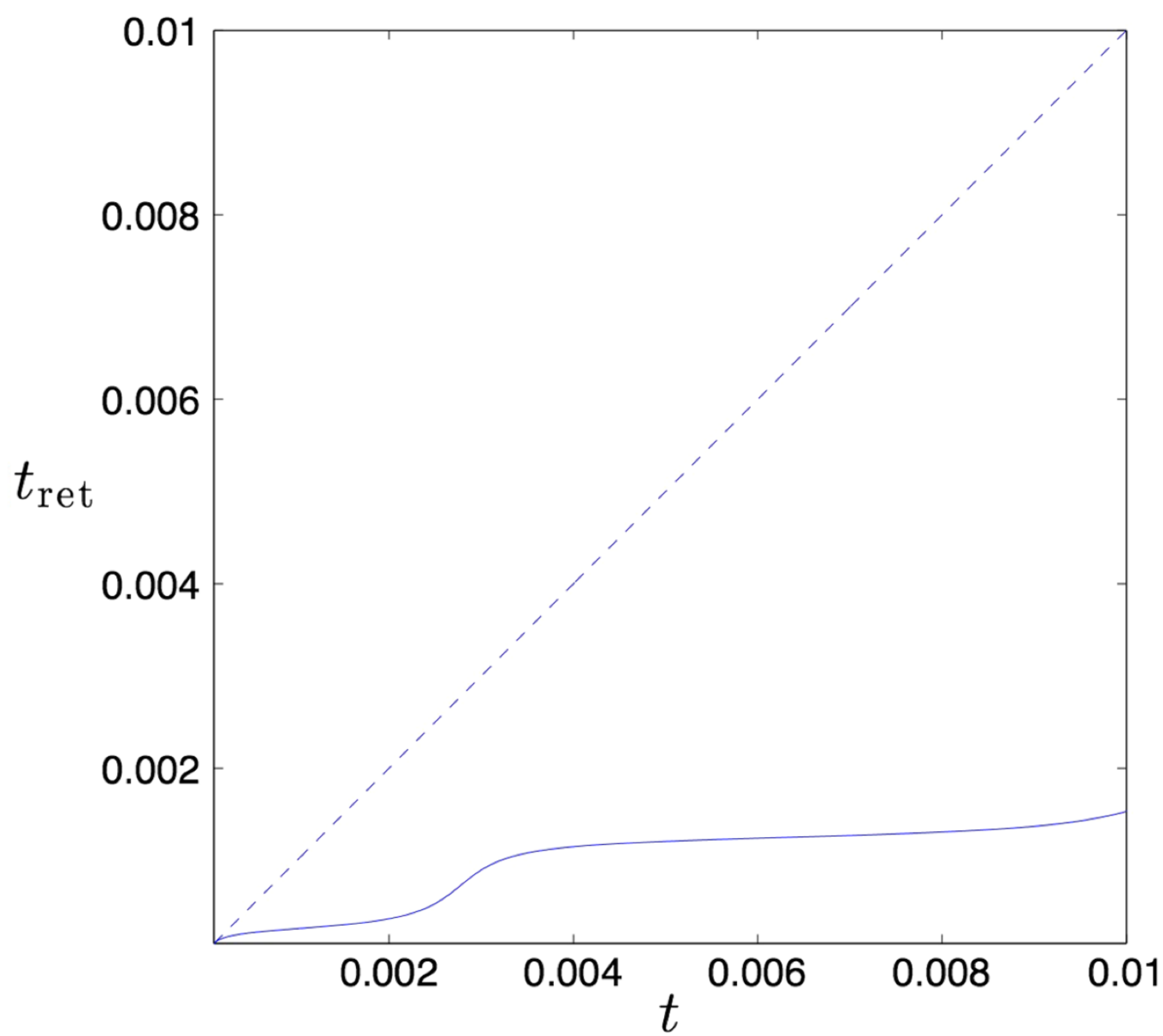
comes from 0^{th} order

Out-of-equilibrium time evolution

- Usual behaviour = evolves towards equilibrium (Minkowski or slowly expanding Universe)

- Inflation: there is a retarded time…



Freezing the pdf out of equilibrium

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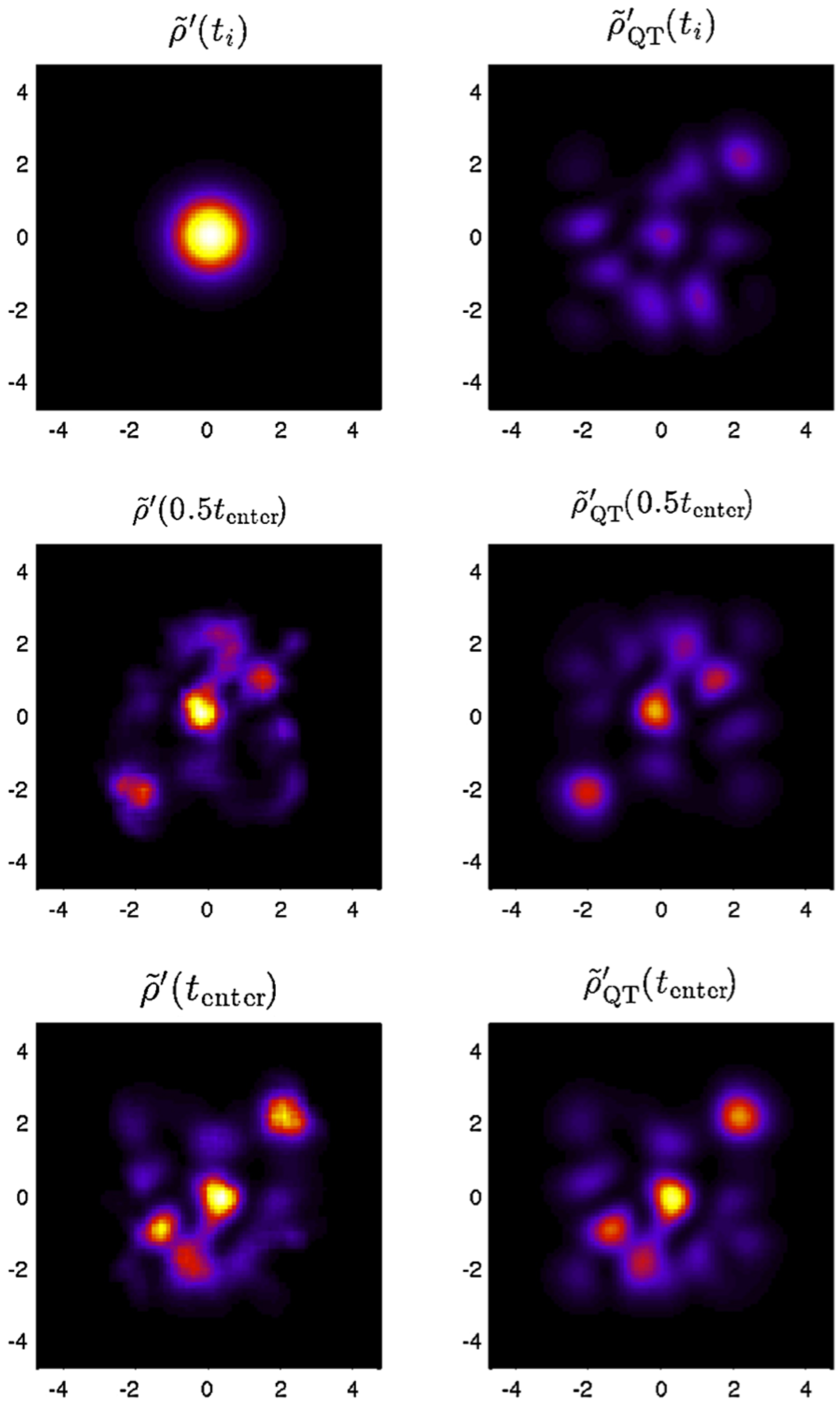
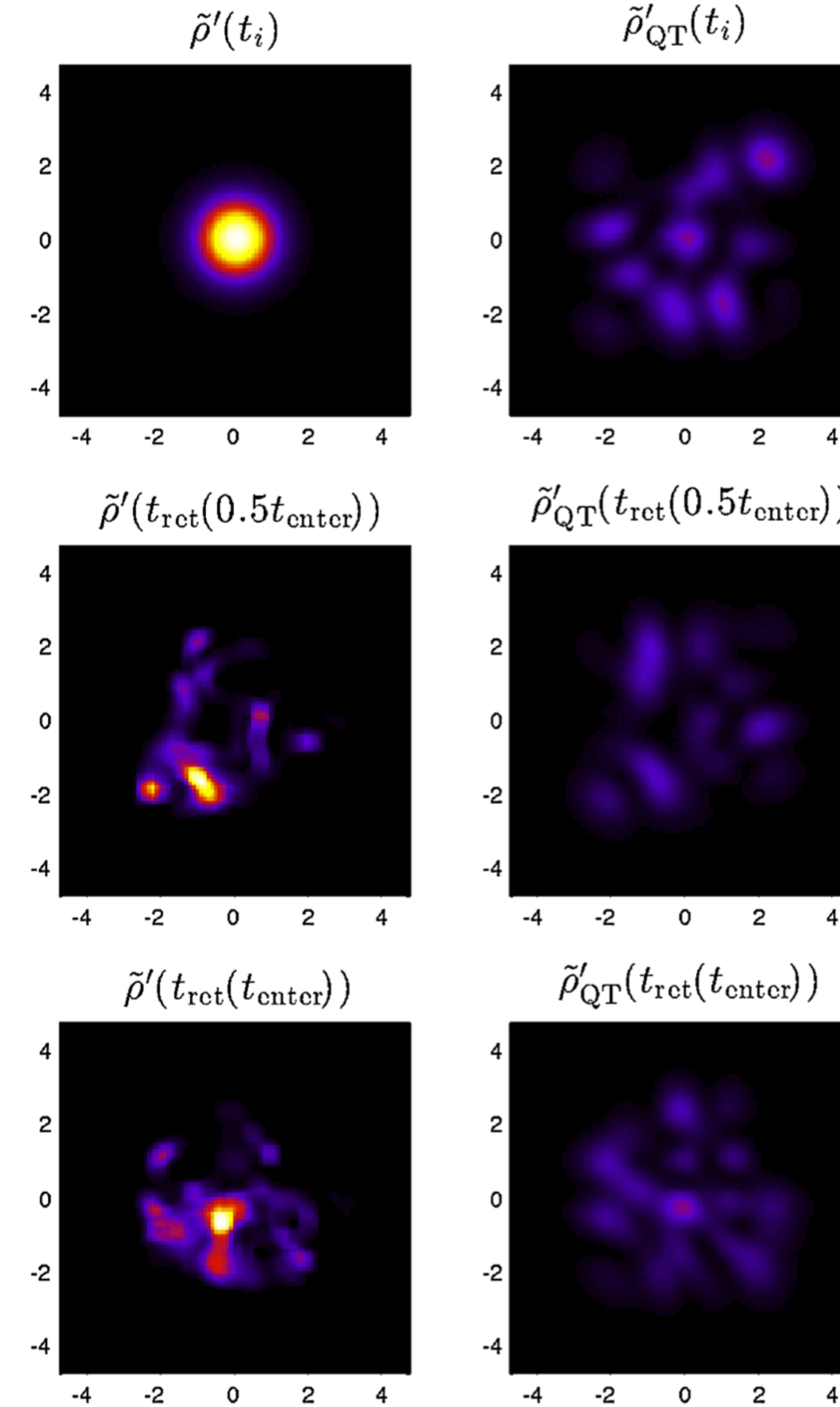
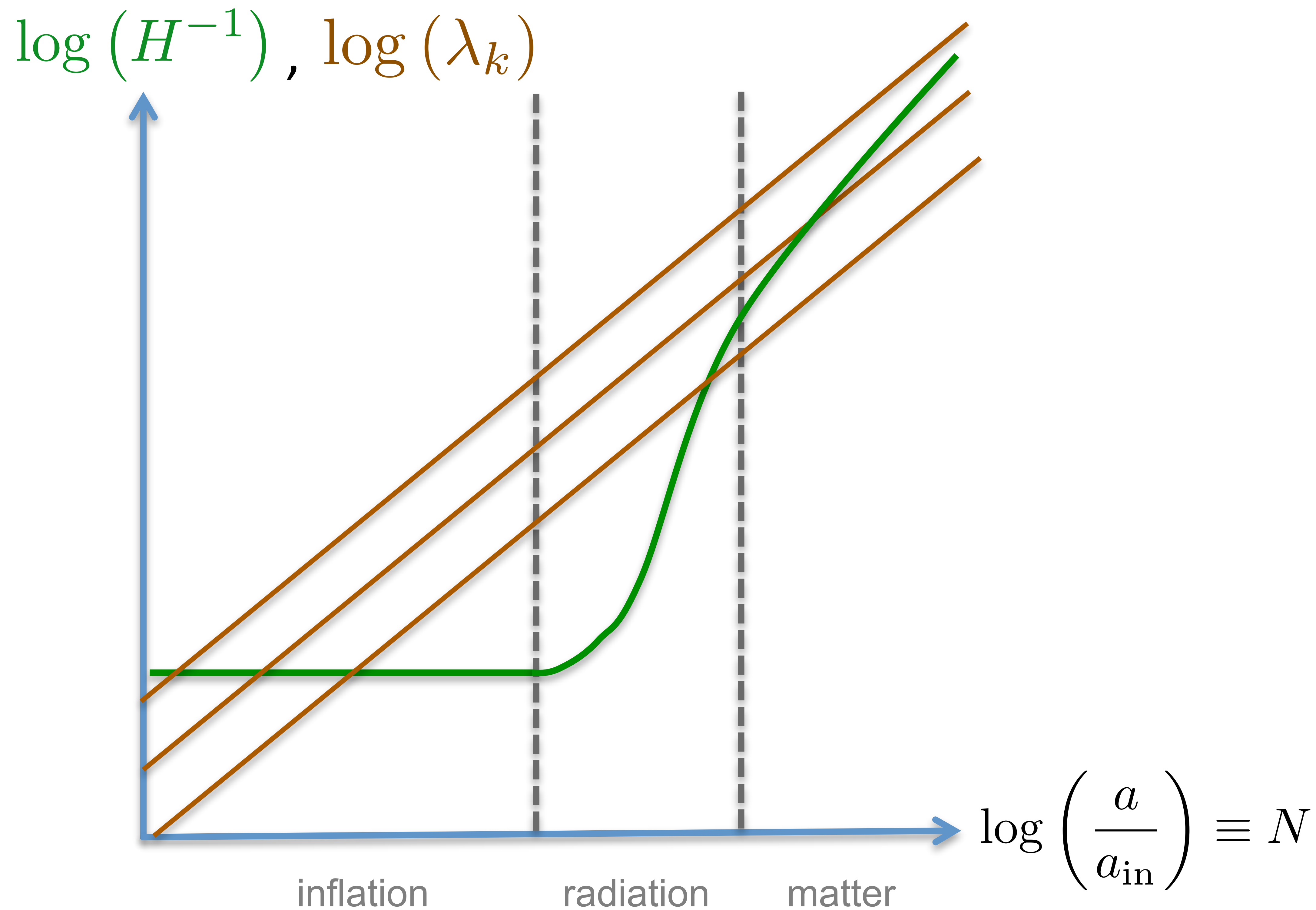


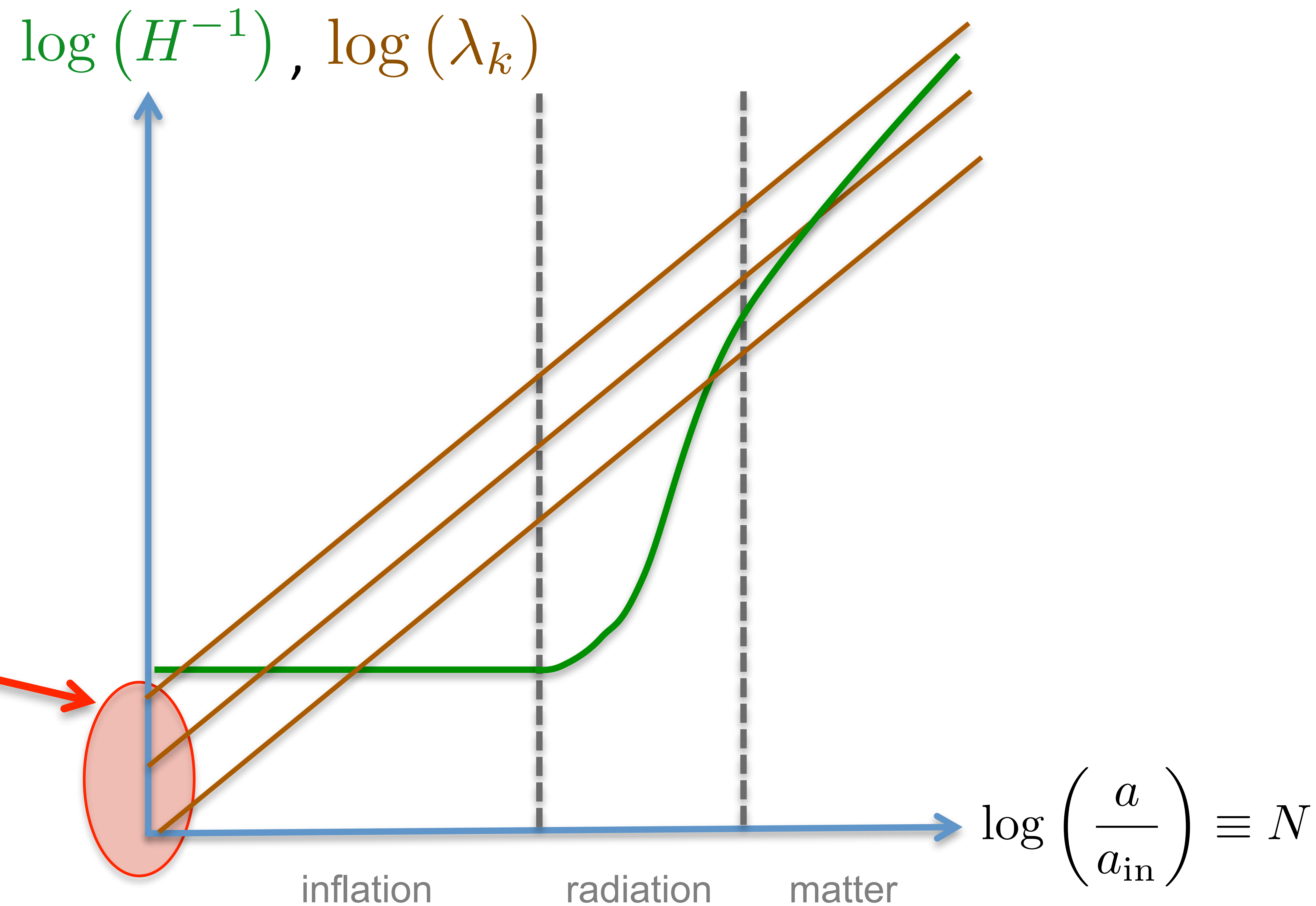
without expansion
EQUILIBRIUM



with expansion
OUT OF EQUILIBRIUM



$\log\left(H^{-1}\right)$, $\log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter



Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

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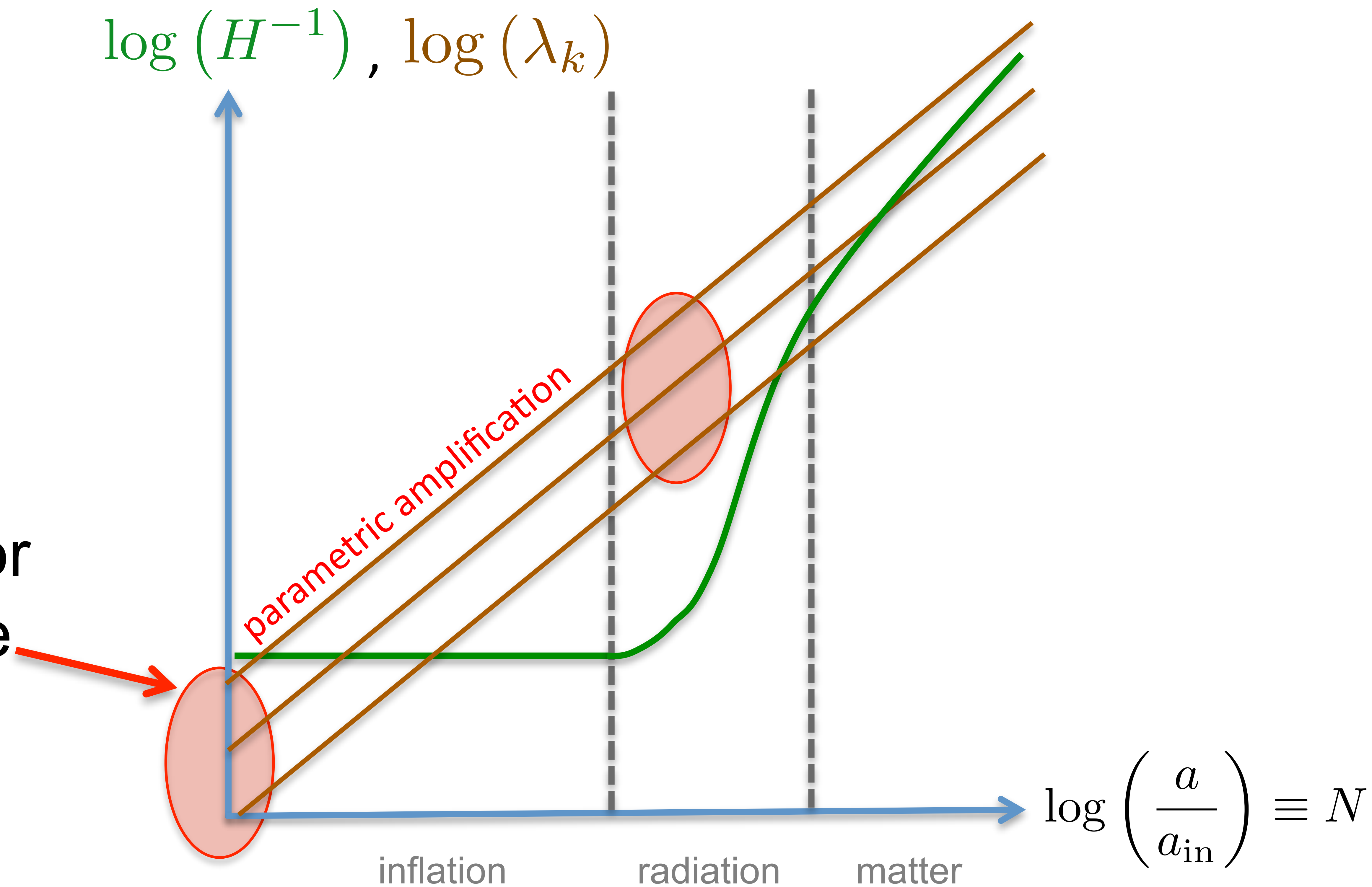

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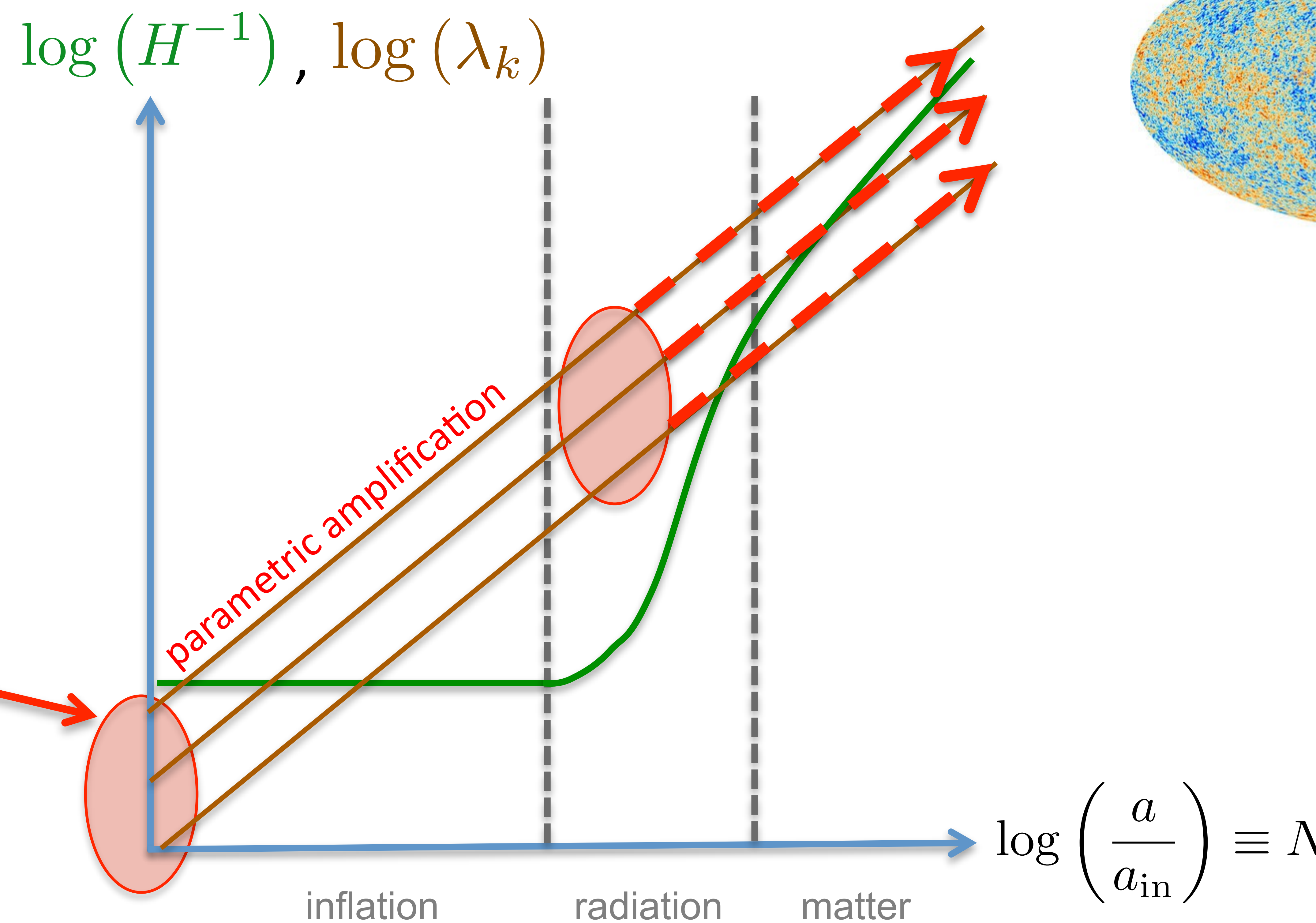

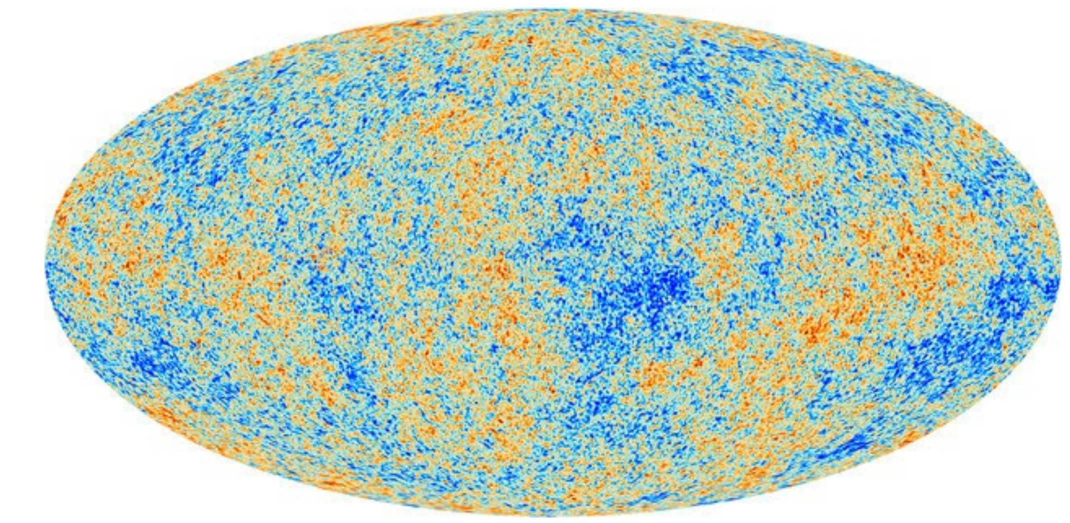

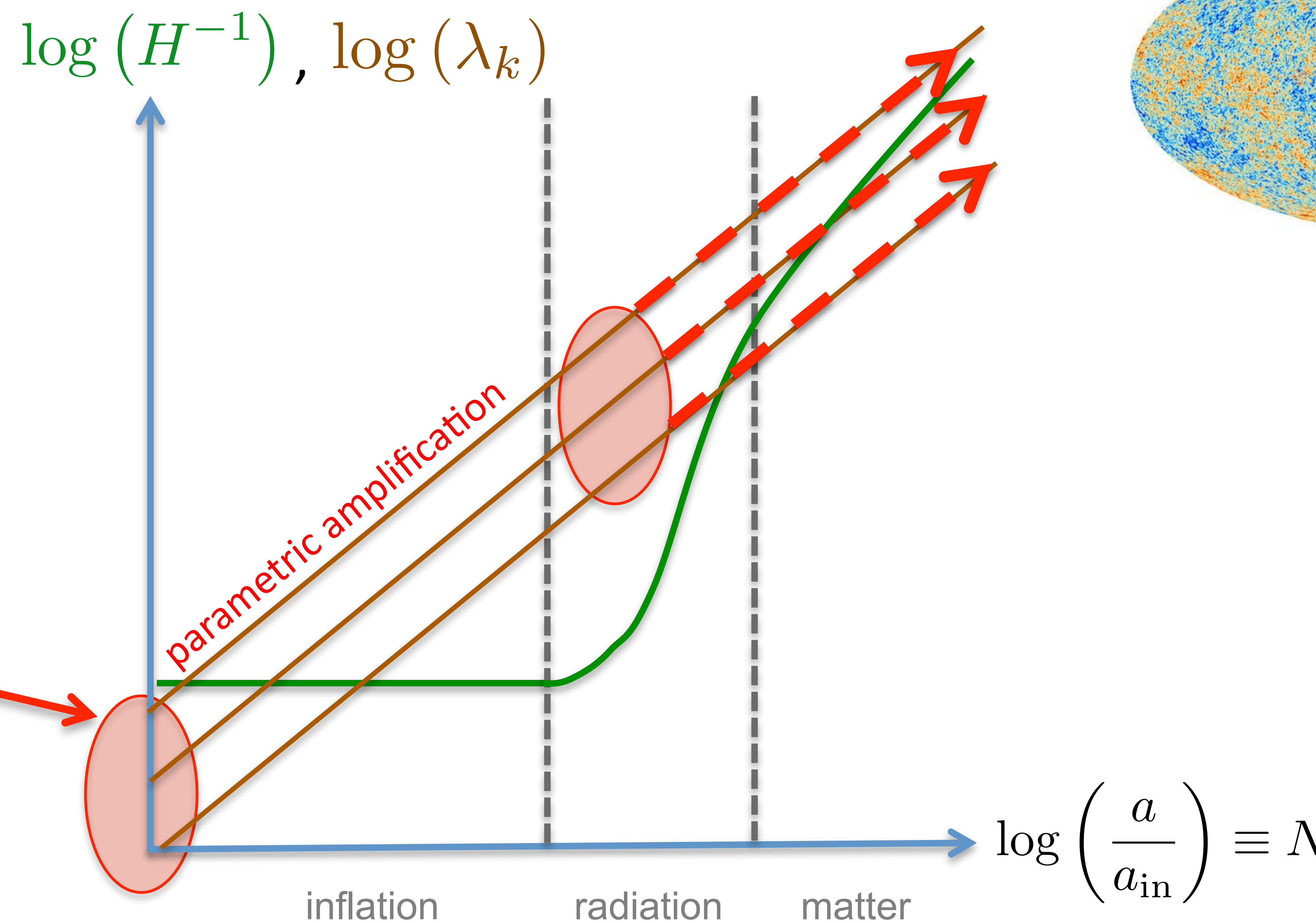

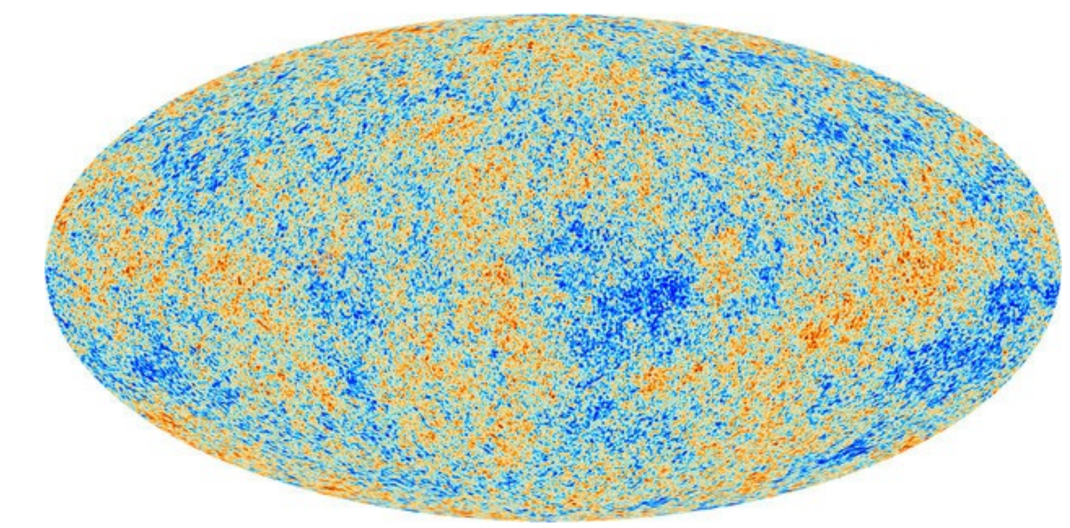

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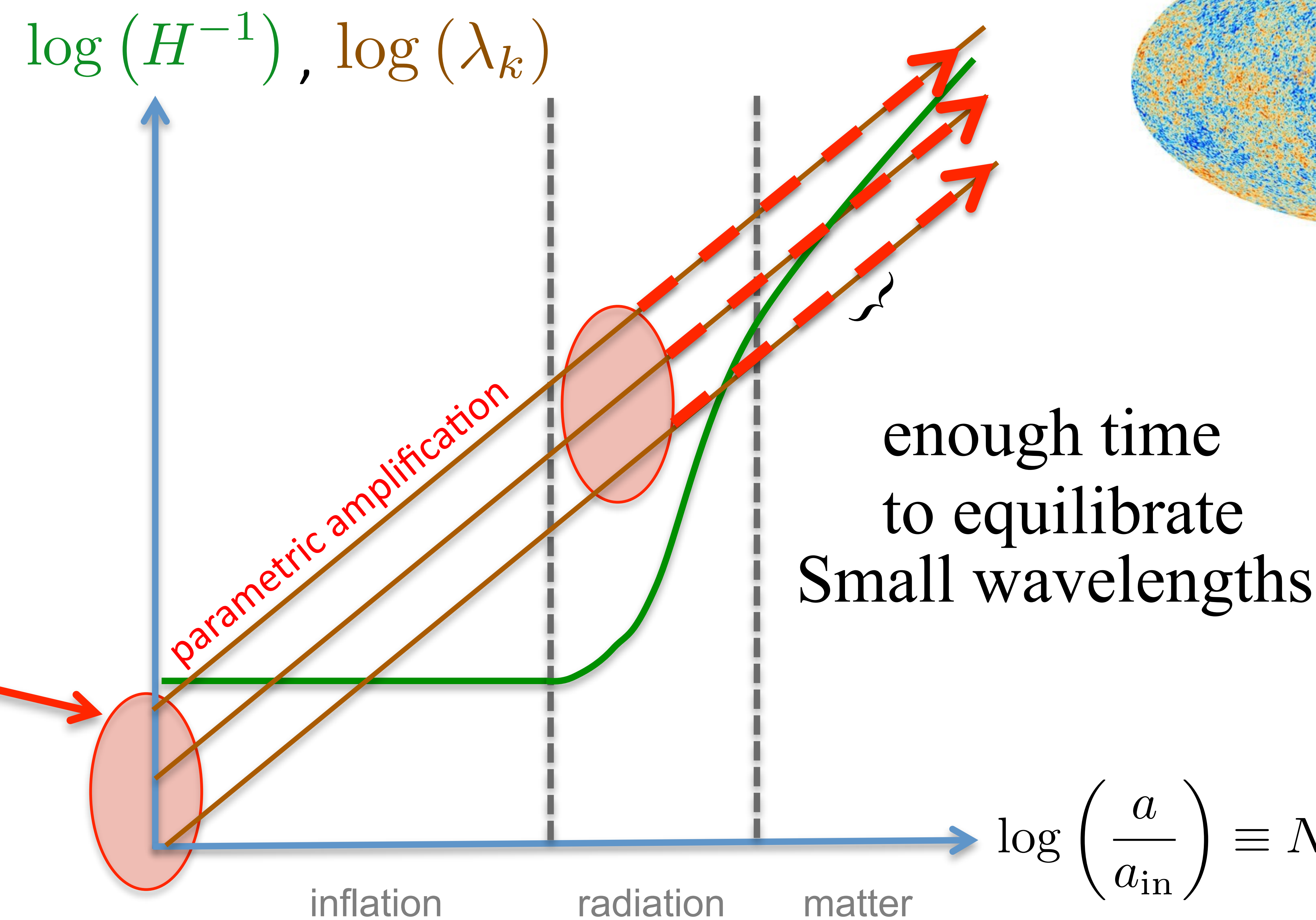
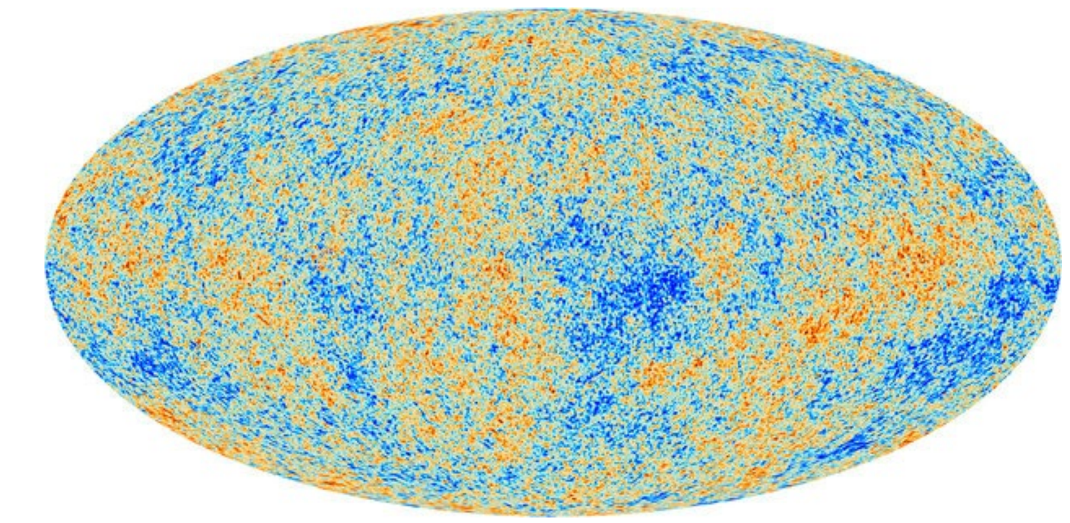
Out-of-Equilibrium
initial density:
less quantum noise





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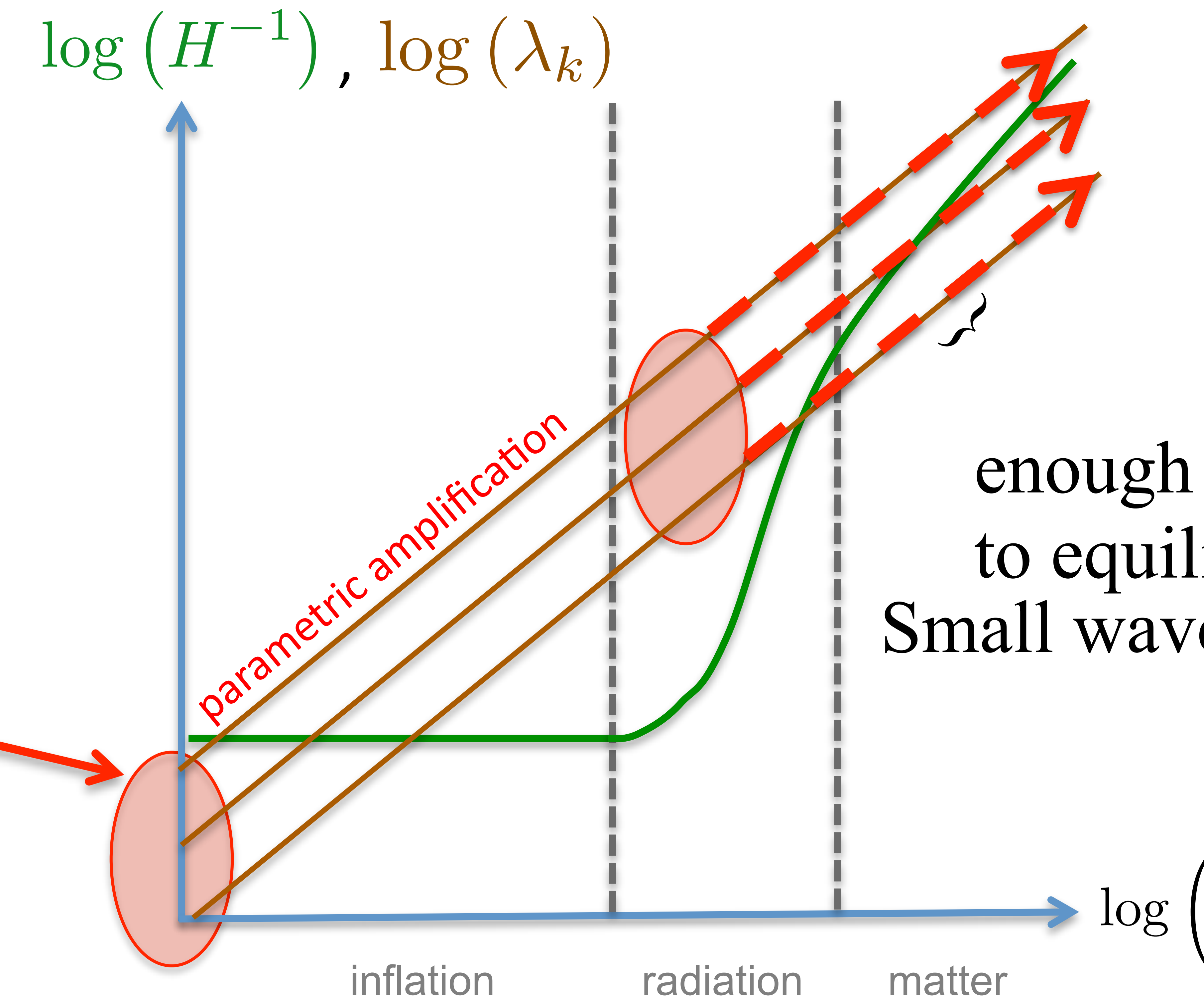

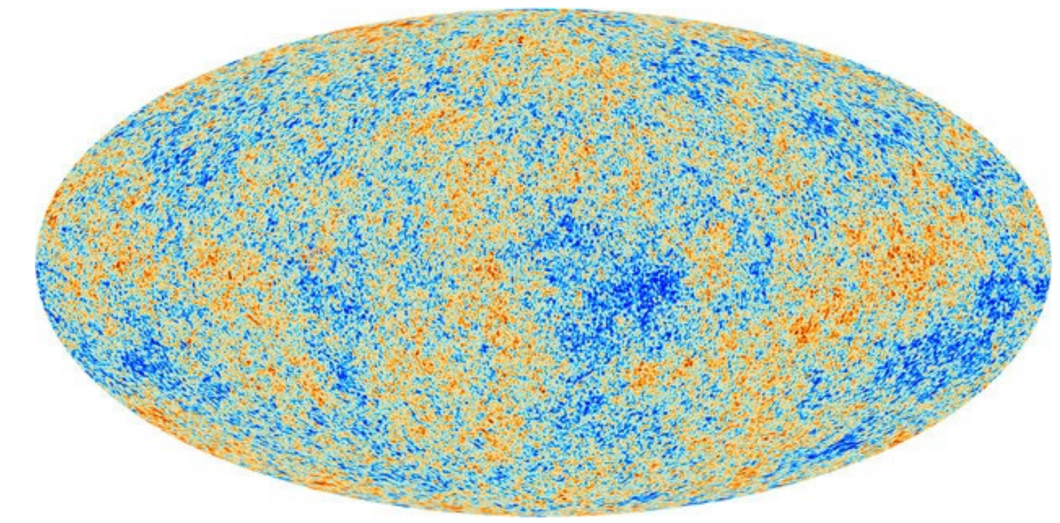

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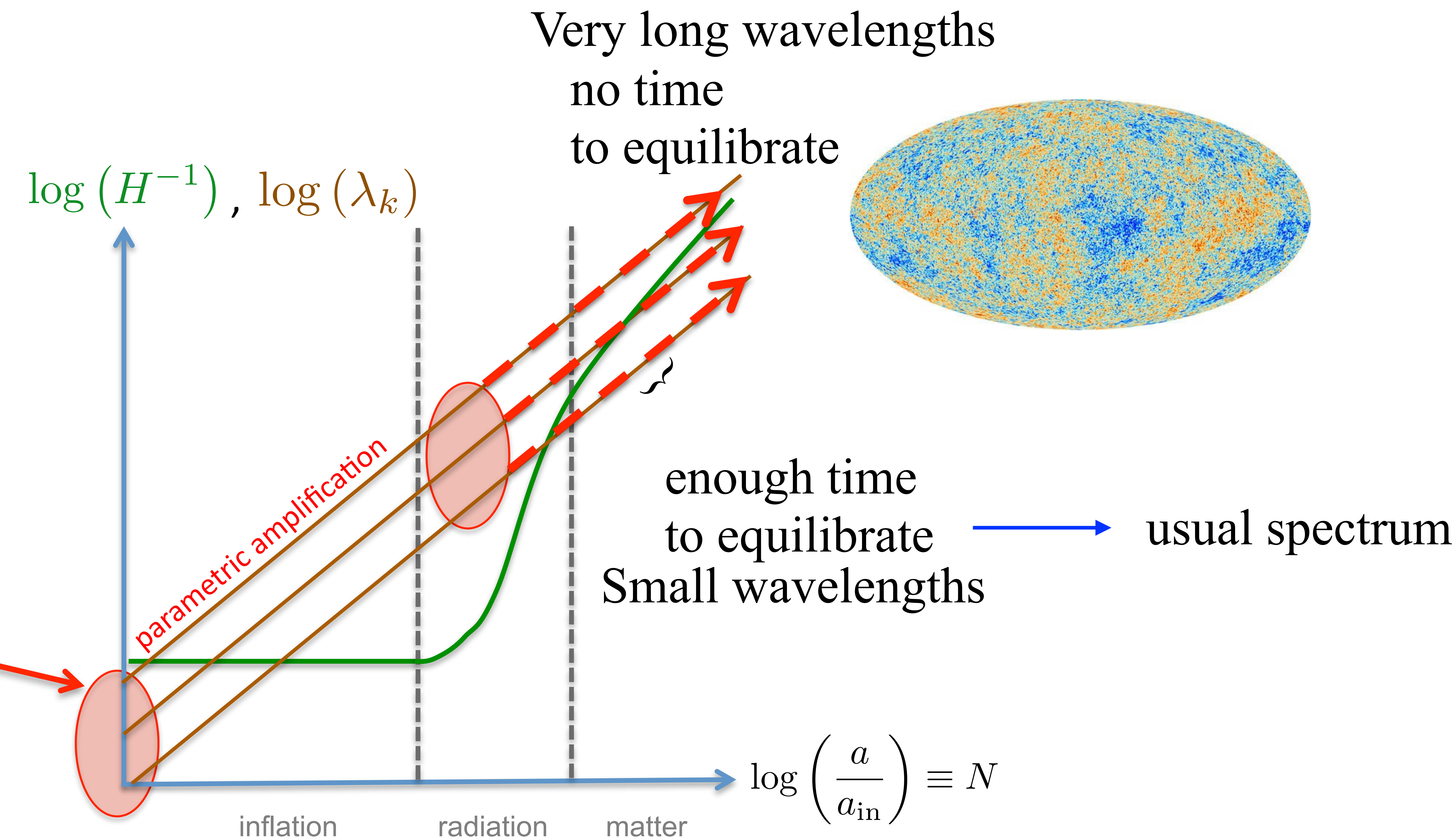

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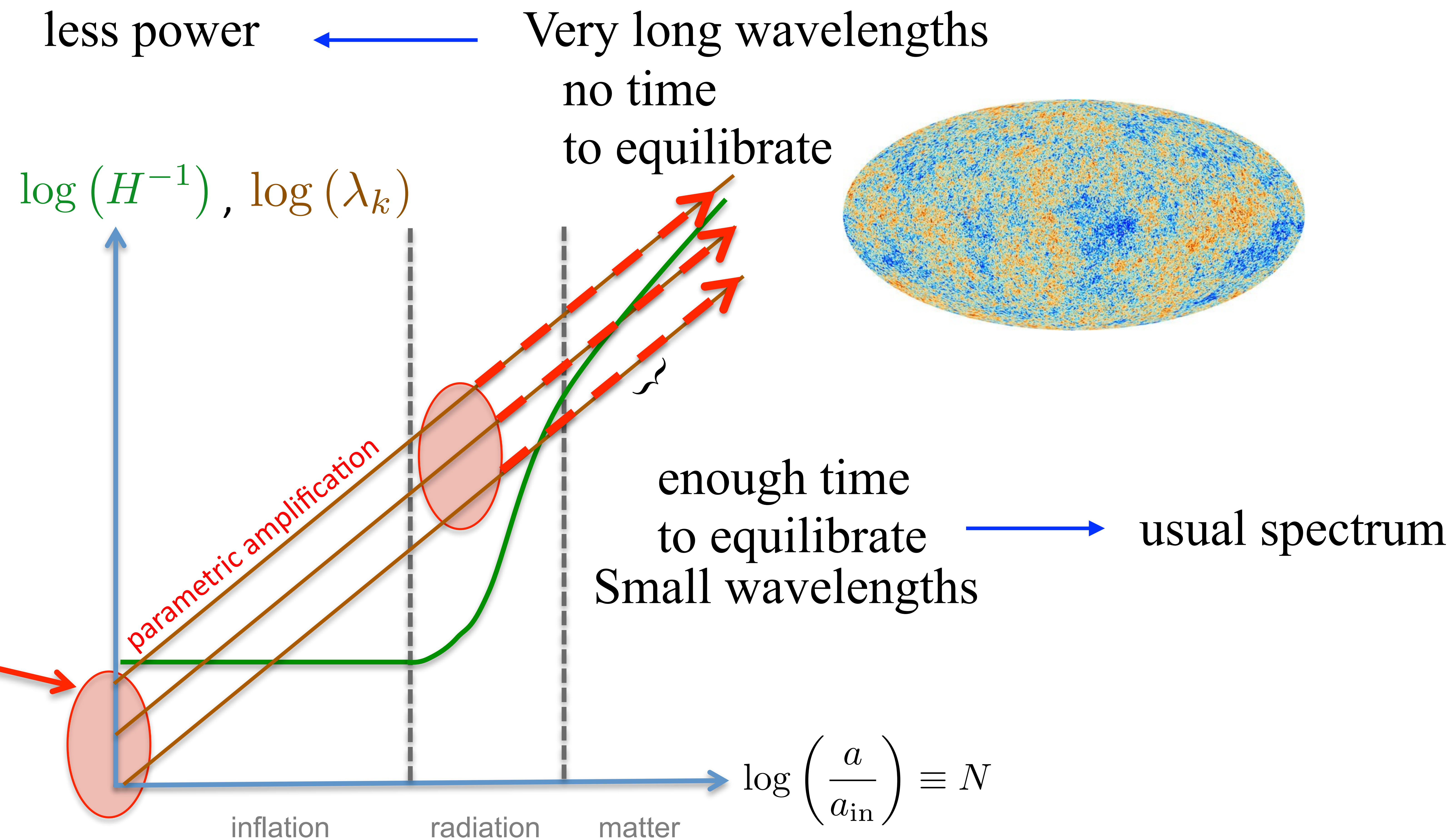

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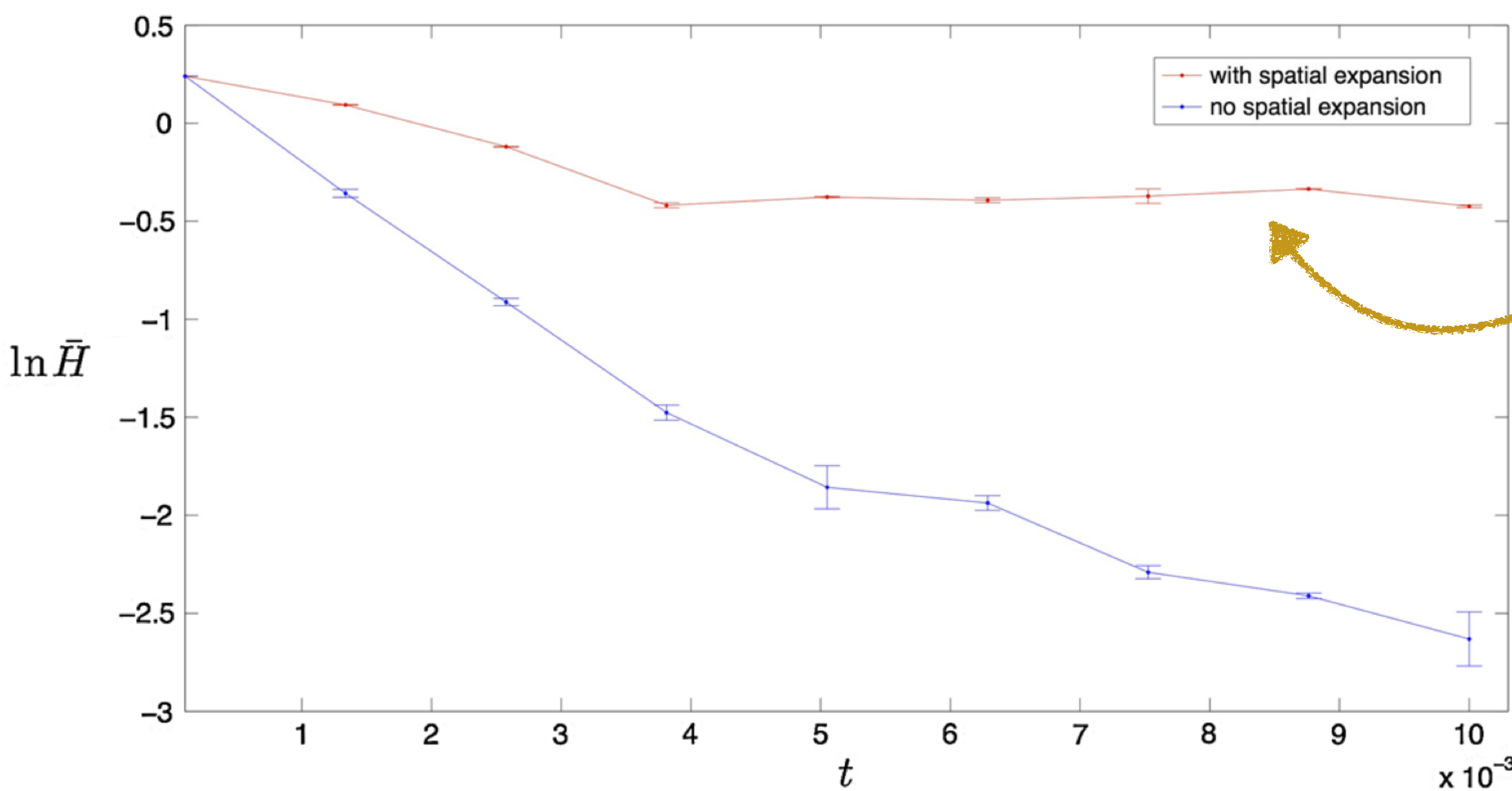
$$H \equiv \int \mathrm{d}q \, \rho \ln \left(\frac{\rho}{|\Psi|^2} \right)$$

measures “out-of-equilibrium-ness”

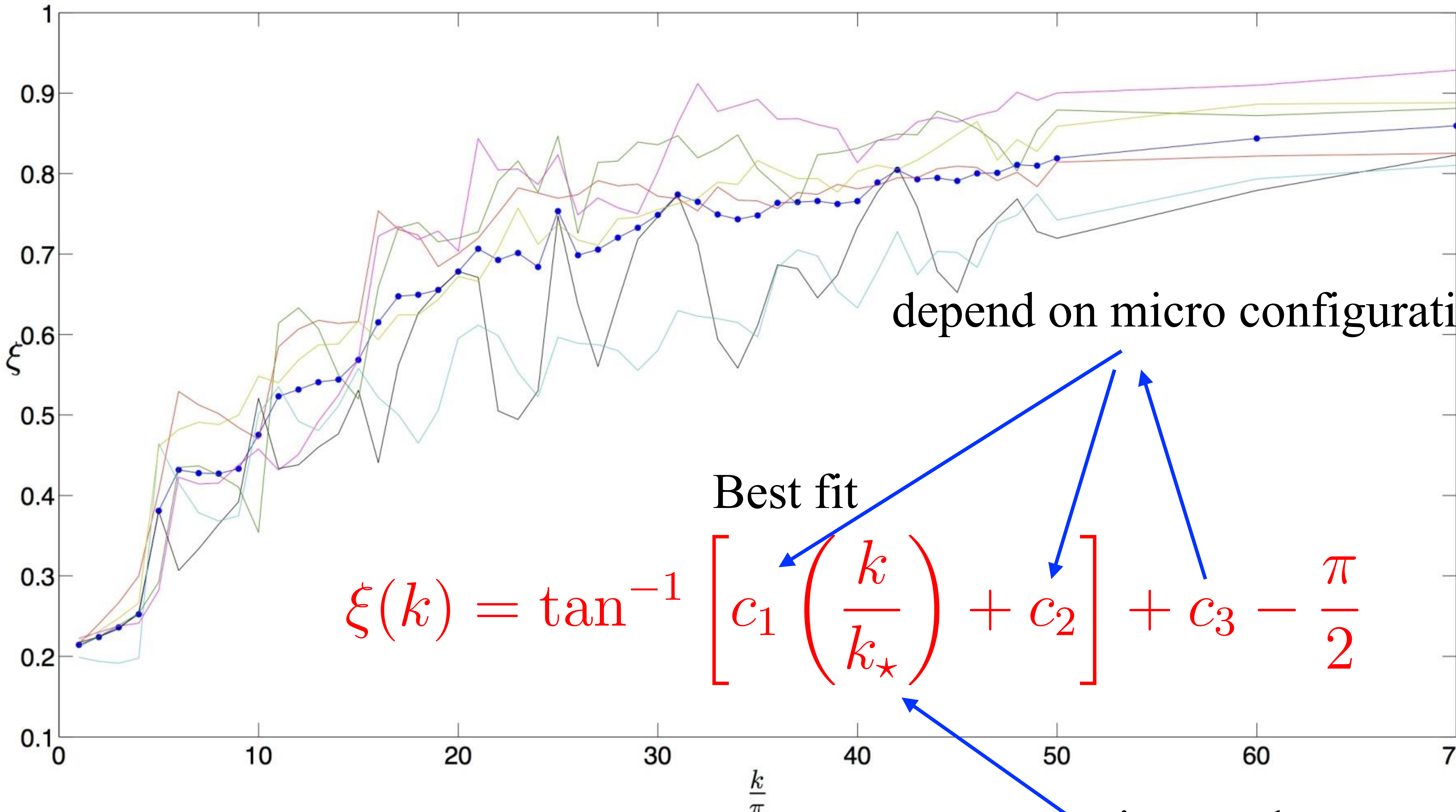


freezing of out-of-equilibrium modes

Initial out-of-equilibrium conditions

$$\mathcal{P}(k) = \mathcal{P}(k)_{\mathrm{QE}}\,\xi(k)$$

width deficit

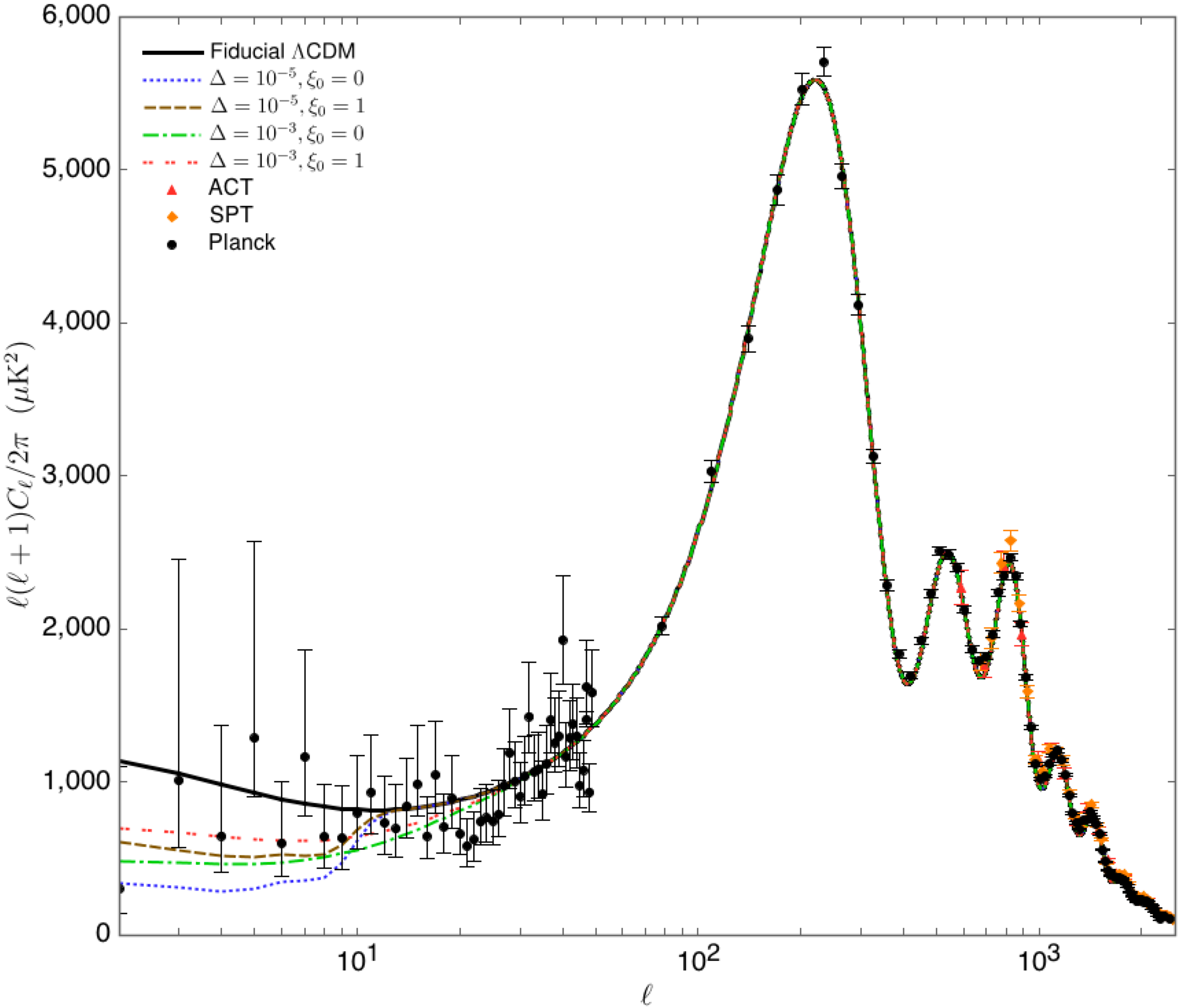


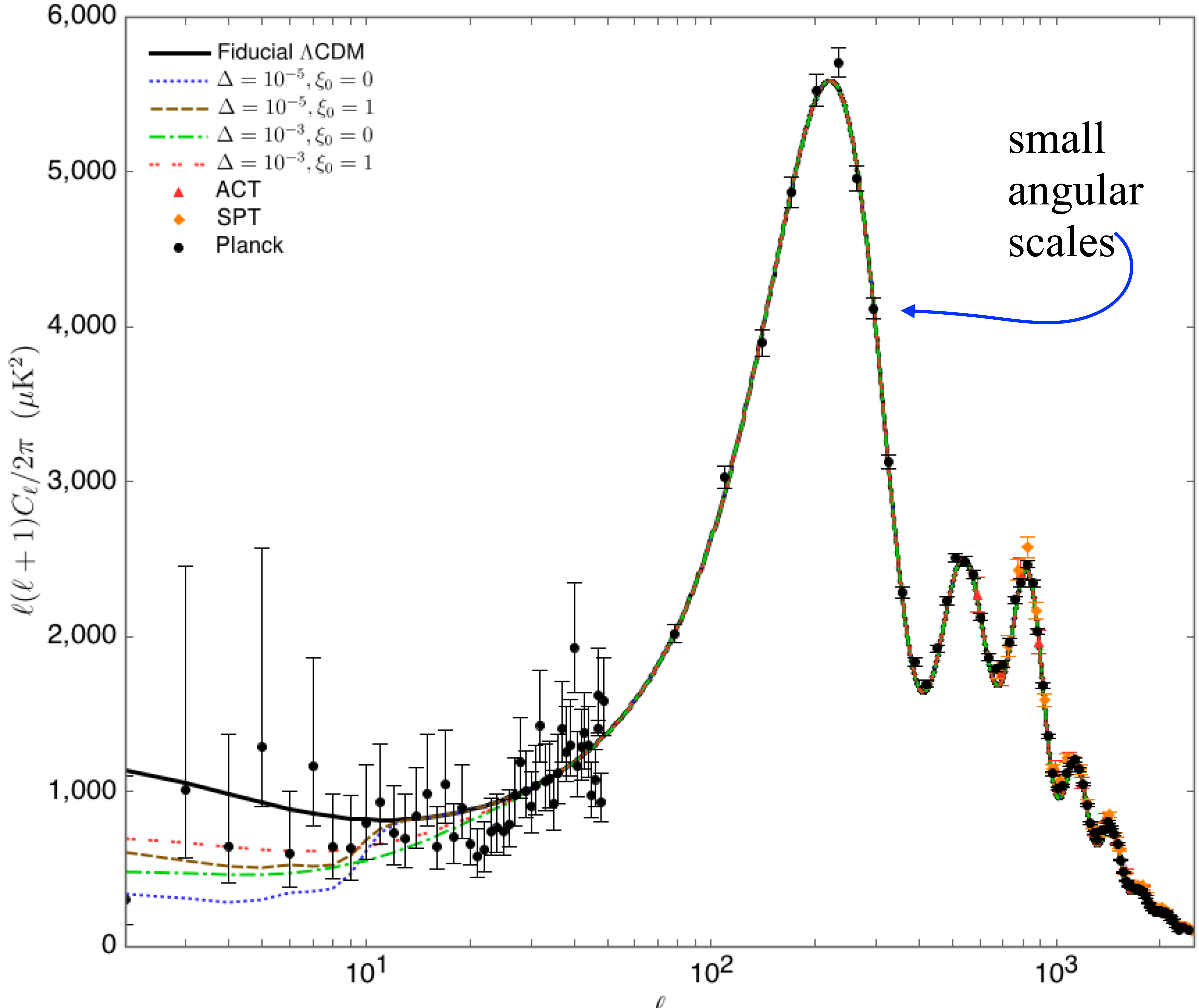
depend on micro configuration

Best fit

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

pivot scale


Fiducial ΛCDM
$\Delta = 10^{-5}, \xi_0 = 0$
$\Delta = 10^{-5}, \xi_0 = 1$
$\Delta = 10^{-3}, \xi_0 = 0$
$\Delta = 10^{-3}, \xi_0 = 1$
ACT
SPT
Planck
6,000
5,000
4,000
3,000
2,000
1,000
0
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
10^1
10^2
10^3
ℓ



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ACT
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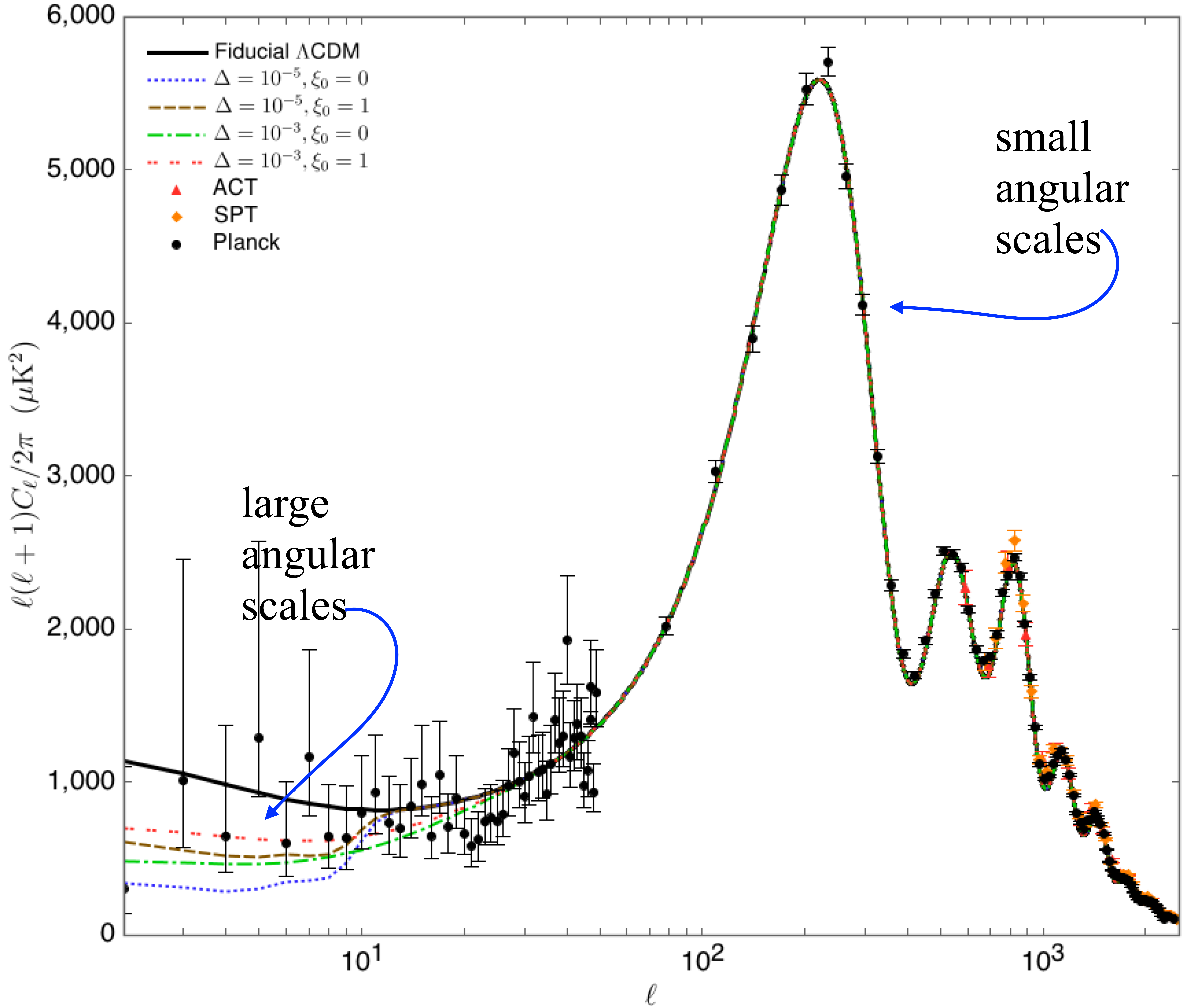
Planck
small angular scales
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
ℓ
6,000
5,000
4,000
3,000
2,000
1,000
0
10^1
10^2
10^3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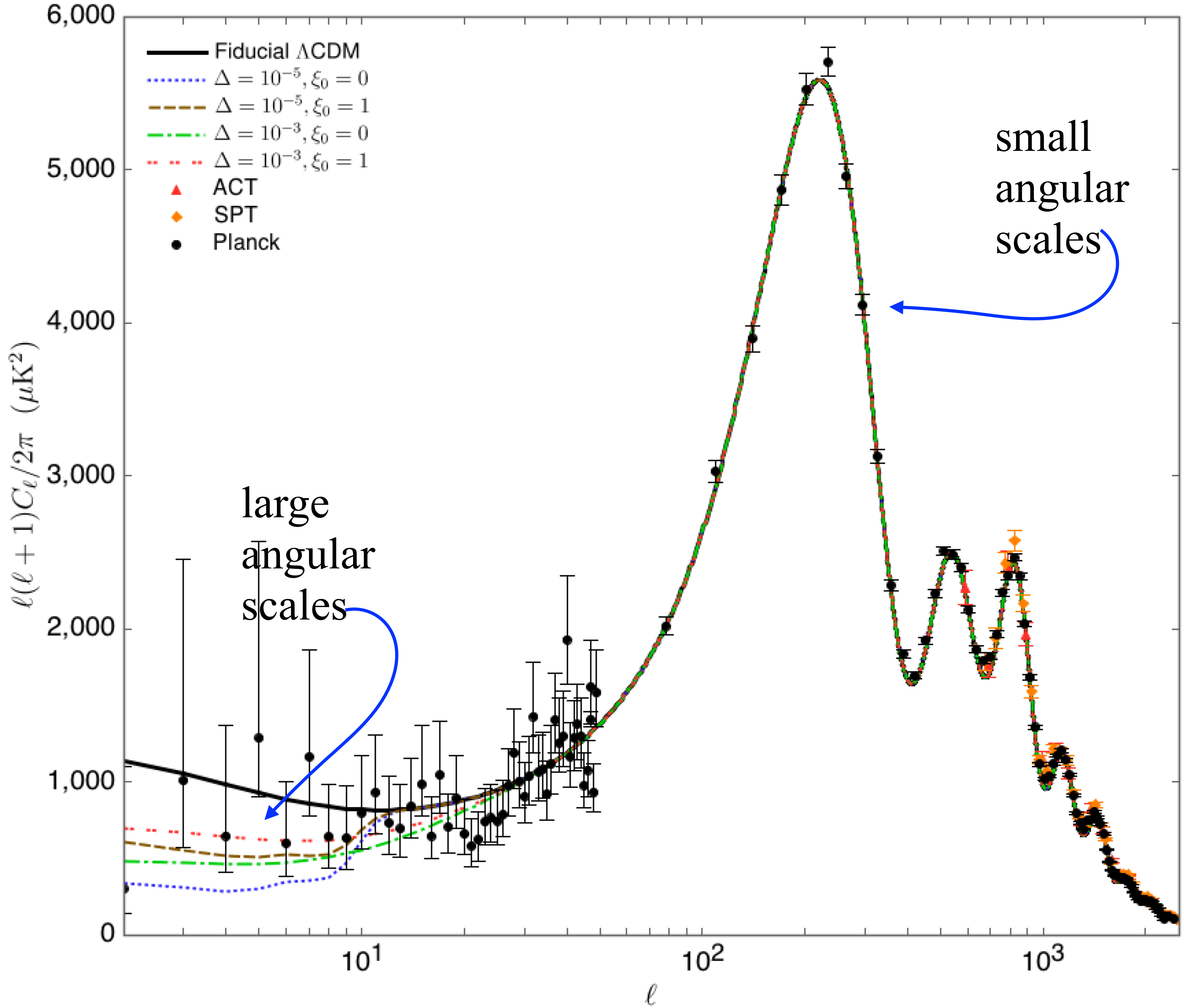
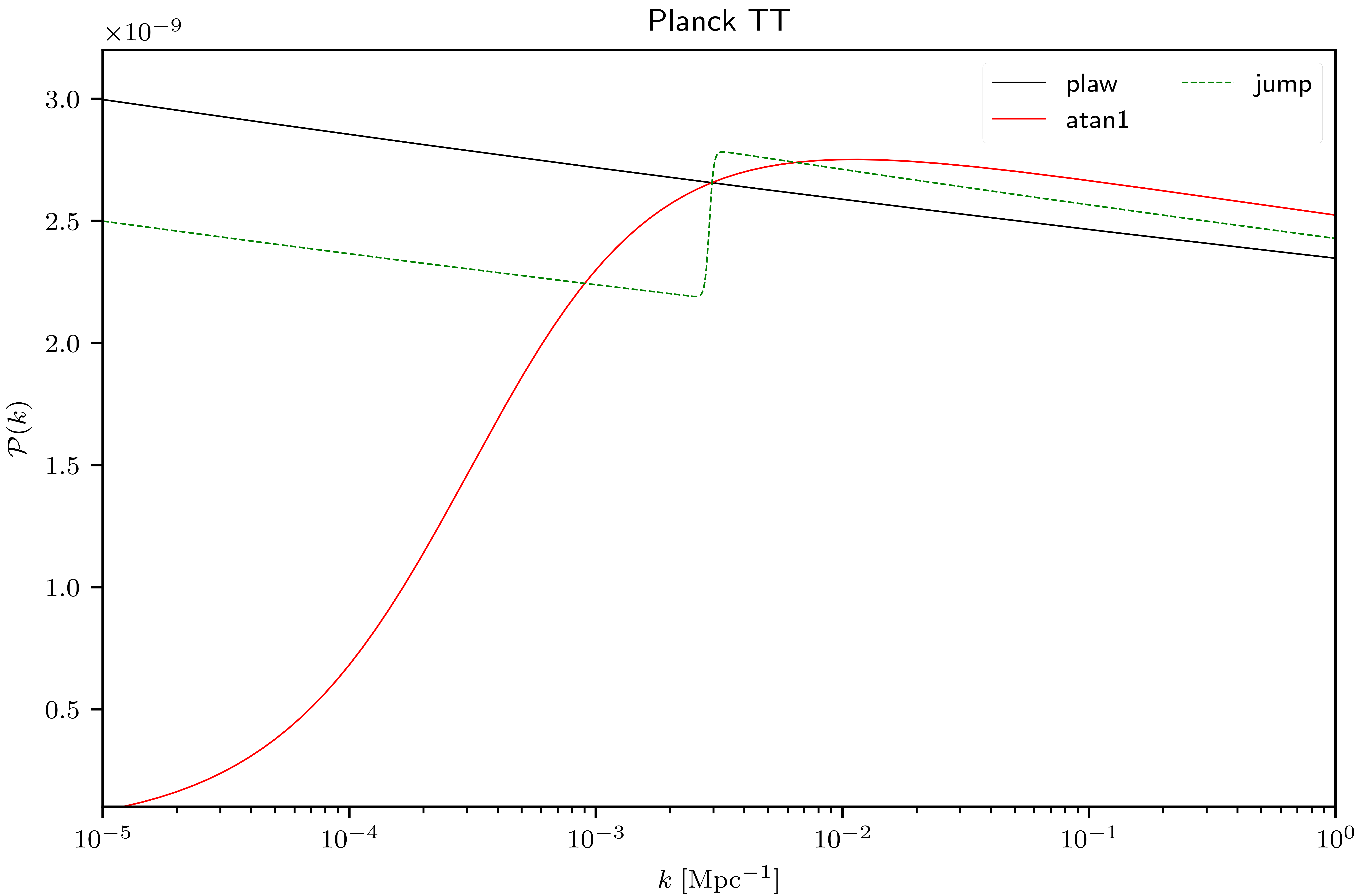
ACT
SPT
Planck
small angular scales
large angular scales
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
6,000
5,000
4,000
3,000
2,000
1,000
0
10^1
10^2
10^3
ℓ

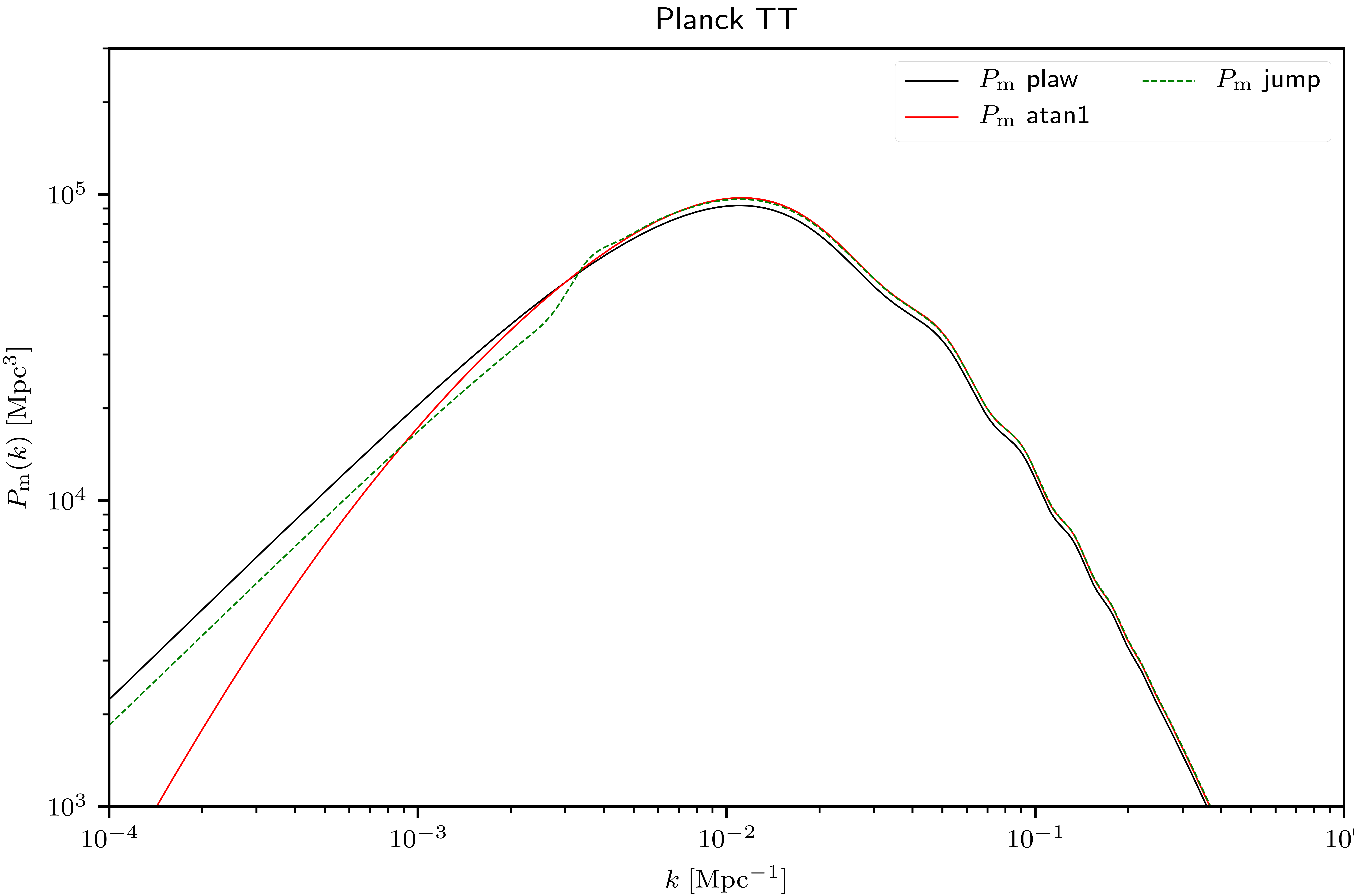


Better fit???

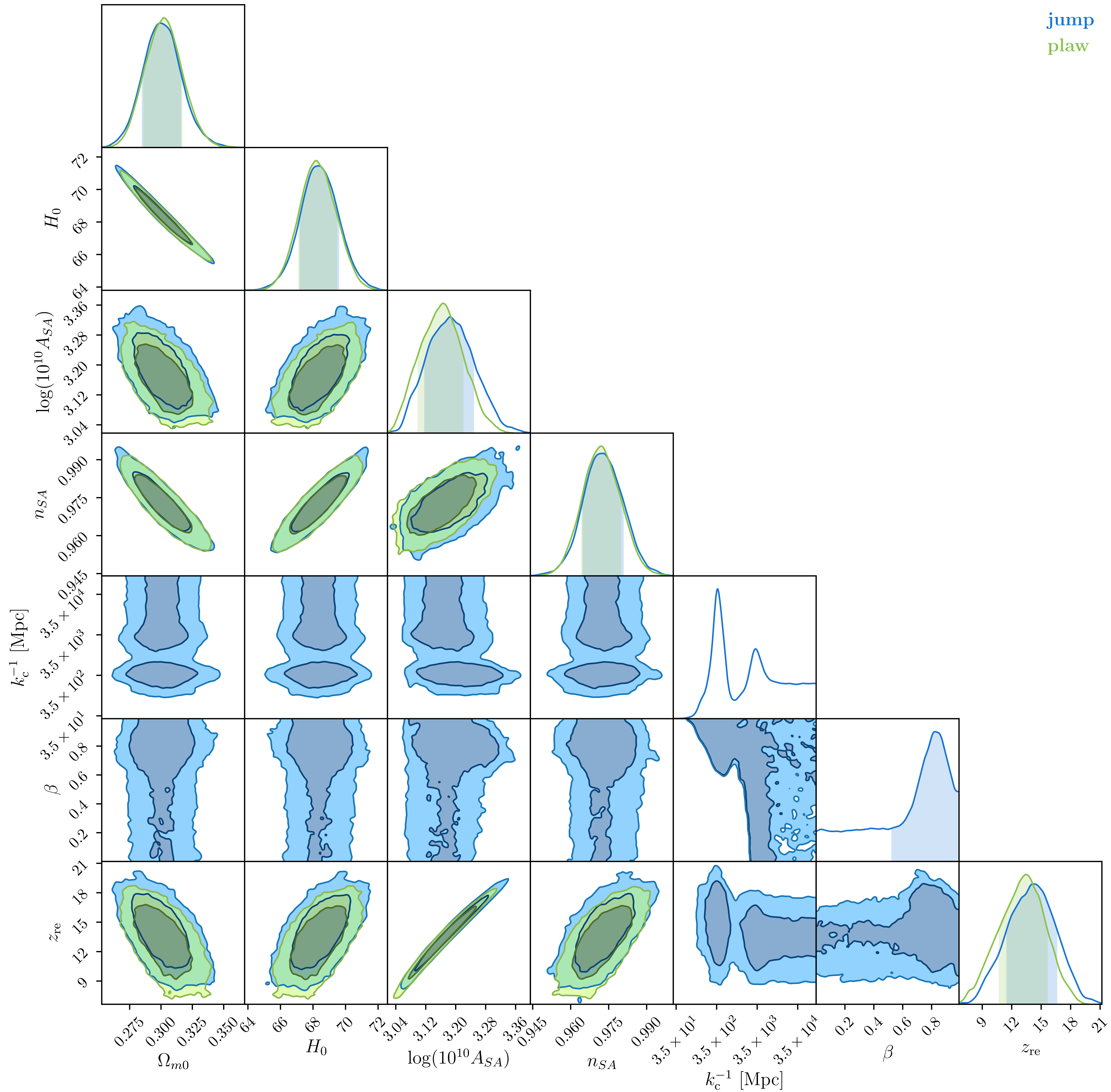
CosmoMC chains



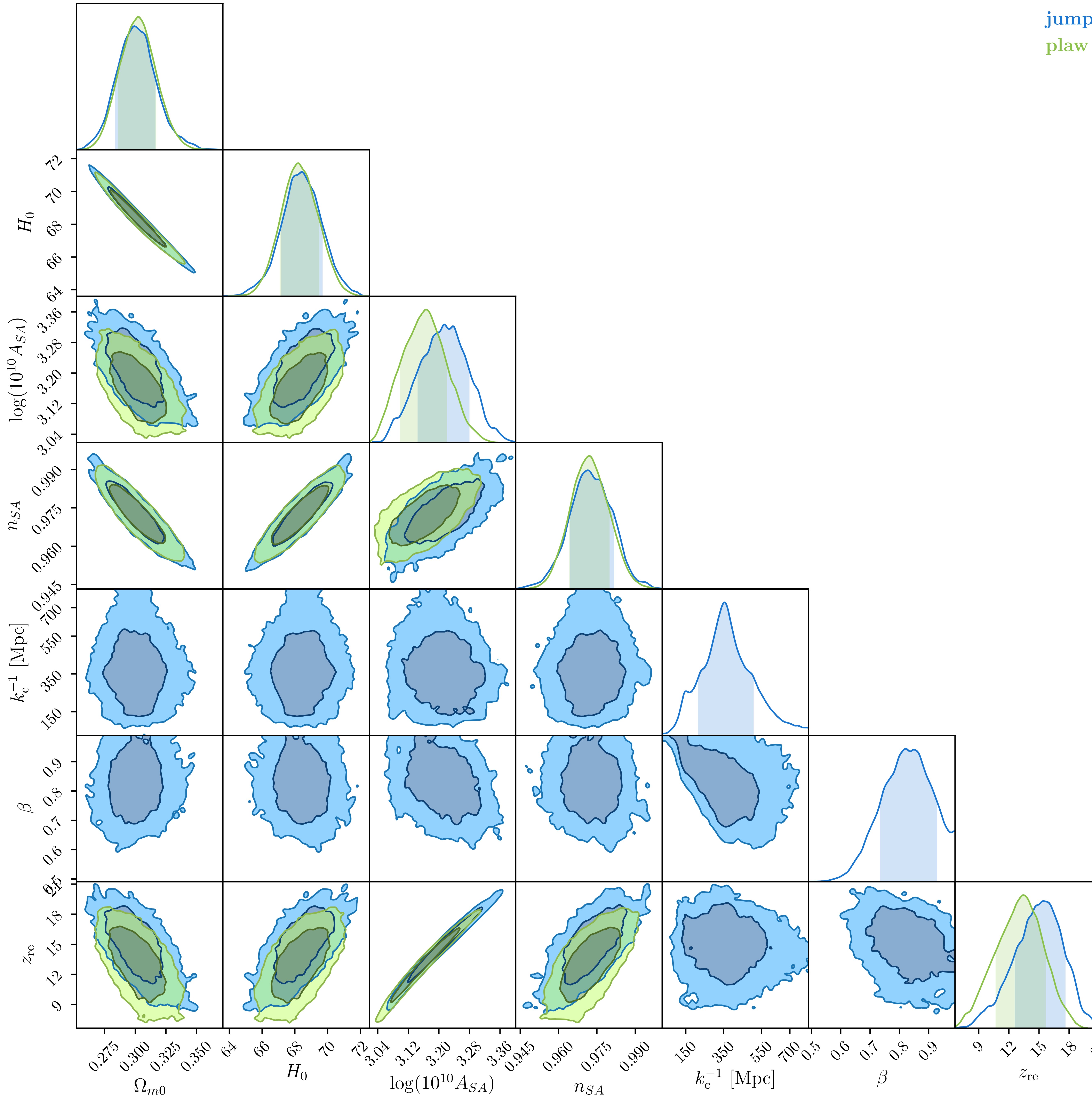
Planck TT
$\times 10^{-9}$
3.0
2.5
2.0
1.5
1.0
0.5
$\mathcal{P}(k)$
plaw
atan1
jump
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
k [Mpc^{-1}]

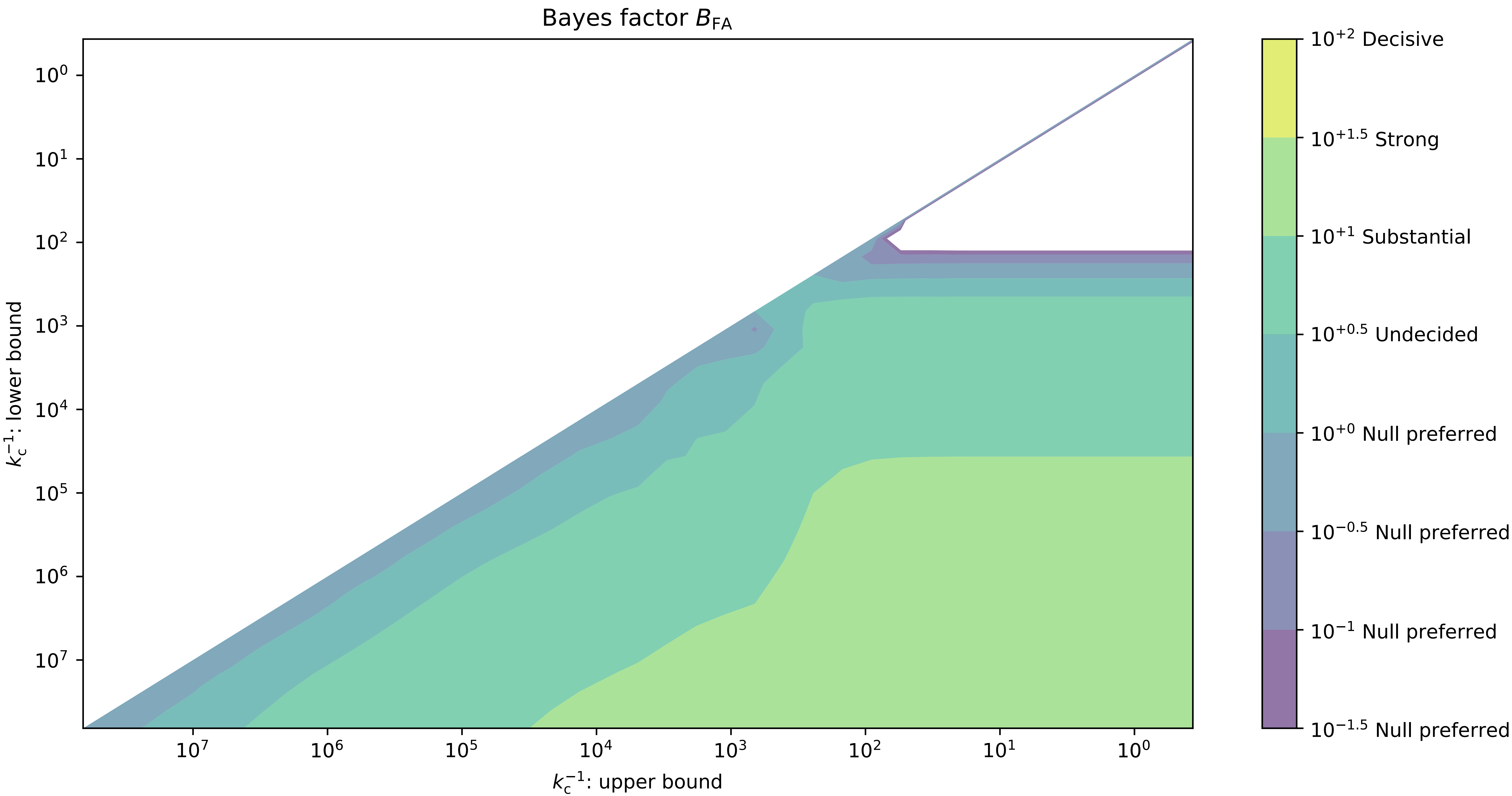


Planck TT
$P_{\rm m}$ plaw
$P_{\rm m}$ atan1
$P_{\rm m}$ jump
$P_{\rm m}(k)$ [Mpc^3]
10^3
10^4
10^5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
k [Mpc^{-1}]



jump
plaw
Ω_{m0}
H_0
$\log(10^{10}A_{SA})$
n_{SA}
k_c^{-1} [Mpc]
β
$z_{\rm re}$


jump
plaw
H_0
$\log(10^{10}A_{SA})$
n_{SA}
k_c^{-1} [Mpc]
β
z_{re}
Ω_{m0}



LUTh - 22/11/2018

CONCLUSIONS